Comments on the Diagnostic Problem

Most of you did the problem well.

If I wrote “try again” on your paper, you made a serious mistake. Please do the problem again. You needn’t turn your new solution in, though I’ll be happy to look at it if you wish.

Since this problem is rather easy, you need to be able to do it well in order to succeed in the course. The homework problems are much harder. See me if you aren’t confident.

Some comments:

(1) To prove something about the orders of elements, one must use the definition of order. A proof that doesn’t refer to the definition can’t be considered complete. It is best to review the definition at the start. This will help to avoid the errors (2) and (3) that are below.

(2) Most of you supposed that \((ab)^n = 1\), and worked to show that \((ba)^n = 1\). This is good, but it shows only that the order of \(ba\) is less than or equal to the order of \(ab\). The proof isn’t complete at that point.

(3) Many of you forgot to mention the case of infinite order. It should be mentioned, though it is very easy.

(4) If you showed that \((ab)^n = 1\) if and only if \((ba)^n = 1\), and said you were done, you are correct. However, it would be best to clarify your reasoning by reference to the definition.

One final comment about proof by contradiction. I call what comes next a fake contradiction argument and I dislike it.

From \((ab)^n = 1\), one finds that \((ba)^n = 1\). Therefore the order of \(ba\) is at most \(n\). Suppose that the order of \(ba\) is \(k\), and that \(n > k\). From \((ba)^k = 1\) one finds that \((ab)^k = 1\). Since the order of \(ab\) is \(n\), this is a contradiction.

There are true proofs by contradiction, but this isn’t one of them. It is much nicer to state things in a positive way:

Suppose that \(ab\) has order \(n\) and that \(ba\) has order \(k\). Then from \((ab)^n = 1\), one finds that \((ba)^n = 1\). Therefore \(n \geq k\). Similarly, from \((ba)^k = 1\) one finds that \((ab)^k = 1\). Therefore \(k \geq n\). So \(n = k\).