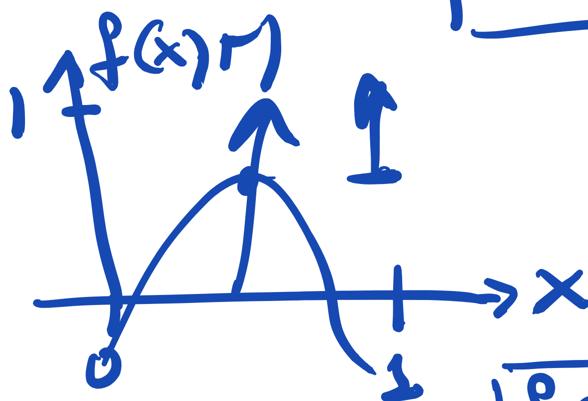
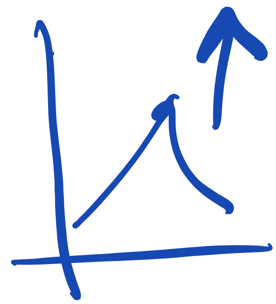
1-D Maps

$$\boxed{x_{n+1} = f(x_n)}$$

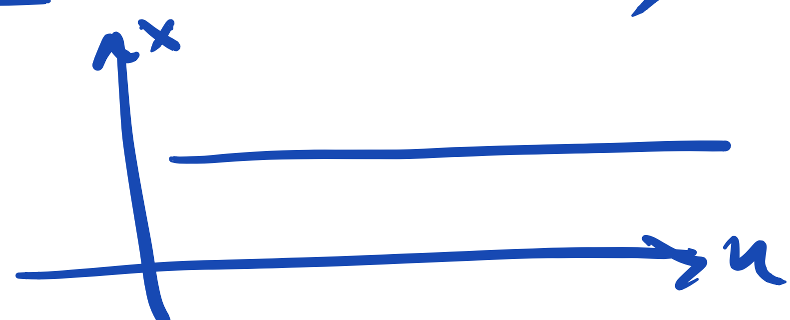




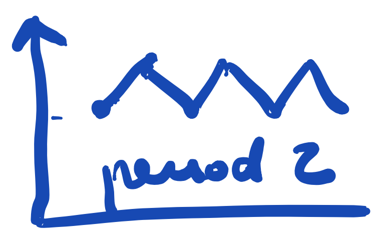
$$\boxed{f(x) = r(1-x)x}$$

Fixed point

$$x_* = f(x_*)$$



periodic orbits



For period 2 $f(f(x_n)) = x_{n+2} = x_n$

period 3 fixed point
$$f_3 = f(f(f(x)))$$

Stability of fixed point

$\boxed{y = f(x)}$ $\qquad x_* = f(x_*)$

$x_{n+1} = f(x_n)$ $\qquad x_n = x_* + \delta_n$

$$\boxed{\delta_{n+1} = f'(x_*)\,\delta_n}$$

$|f'(x_*)| < 1$ [linear stab.

$|f'(x_*)| > 1$ [instability linear

Borderline Case $\quad \underline{\underline{|f'(x_{n})| = 1}}$

$\boxed{\text{Then non-linearity need to ascertain answer}}$

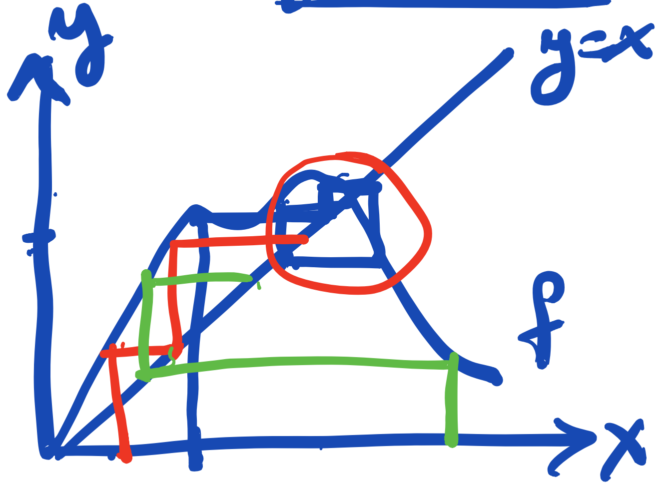
will correspond to **Bifurcations**

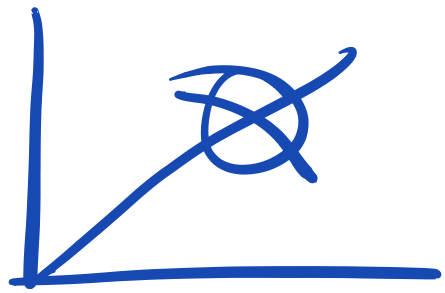
$\underline{\underline{|f'(x)| = 0}}$ $\quad \boxed{\text{Super stable case}}$

Examples and graphs

"Cobweb"

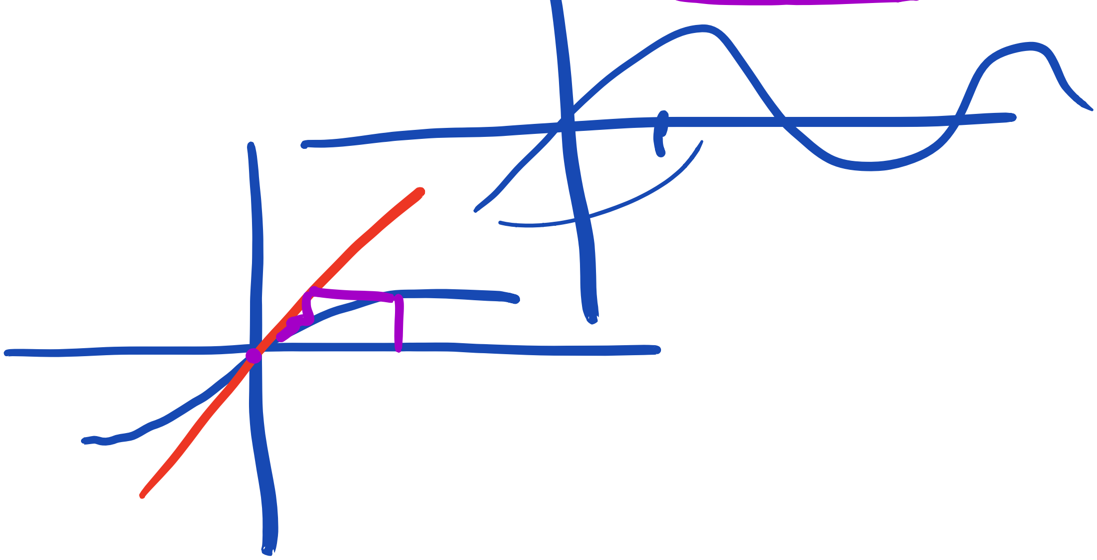
$$\boxed{x_{n+1} = f(x_n)}$$



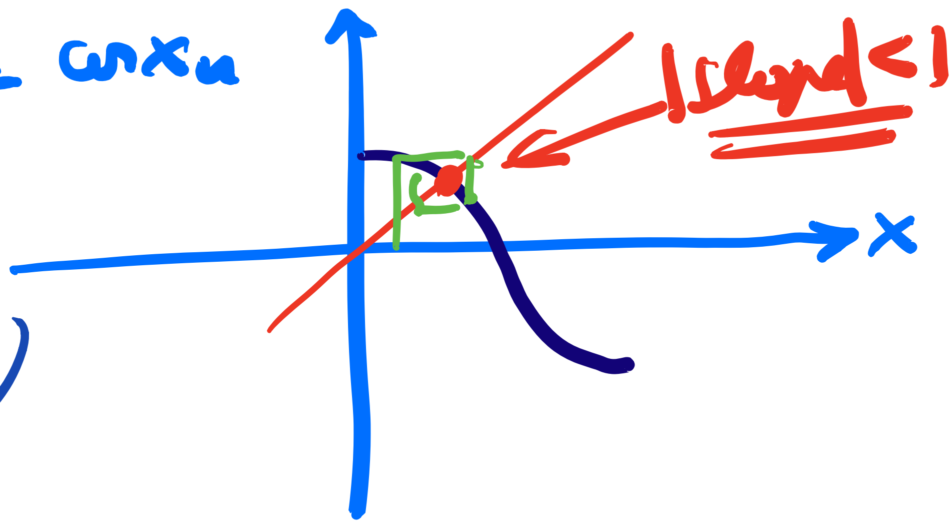


$x_{n+1} = \sin x_n$

$x_n \to 0$



$x_{n+1} = \cos x_n$



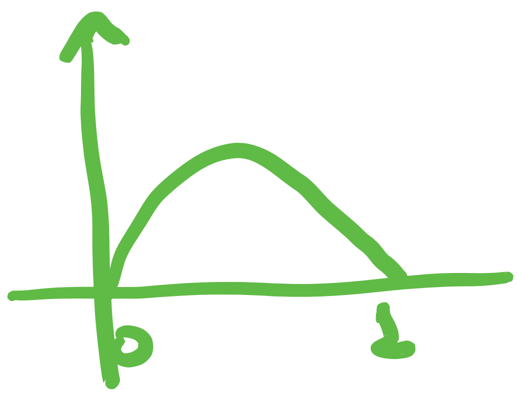
up 359

Logistic map

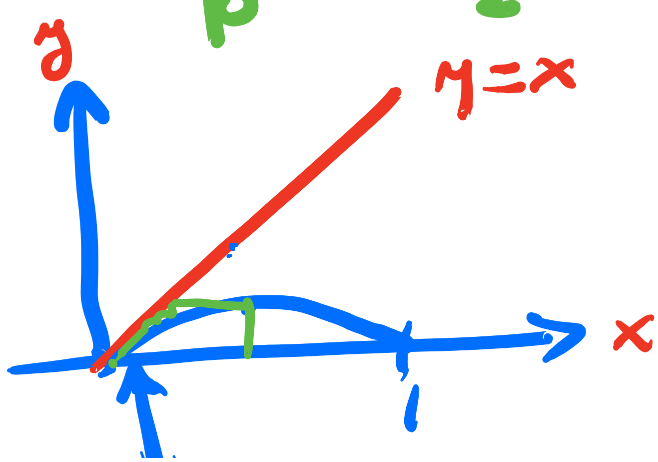
$0 \leq x \leq 1$

$0 \leq r \leq 4$

$$x_{n+1} = r x_n (1 - x_n)$$



$r < 1$

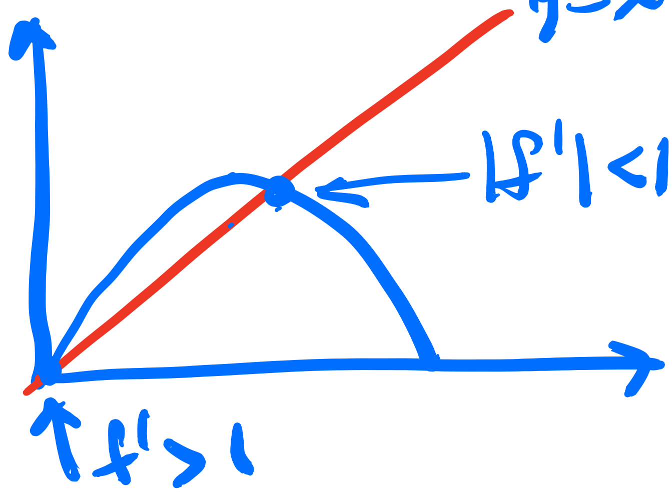


$|f'(0)| < 1$

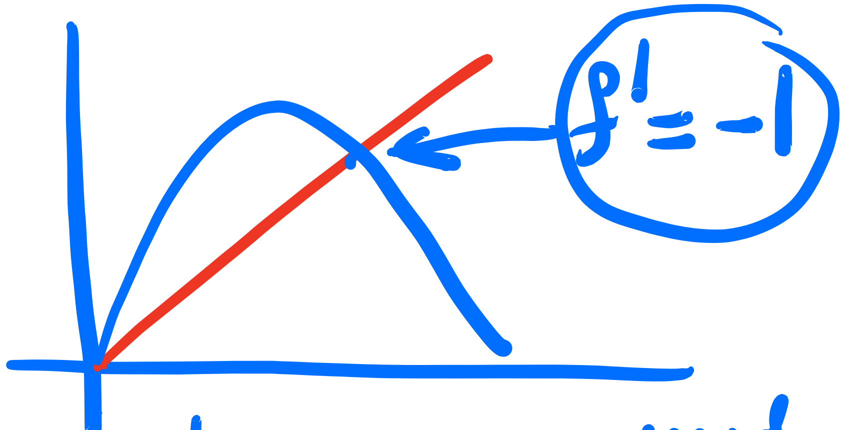
$r > 1$

small

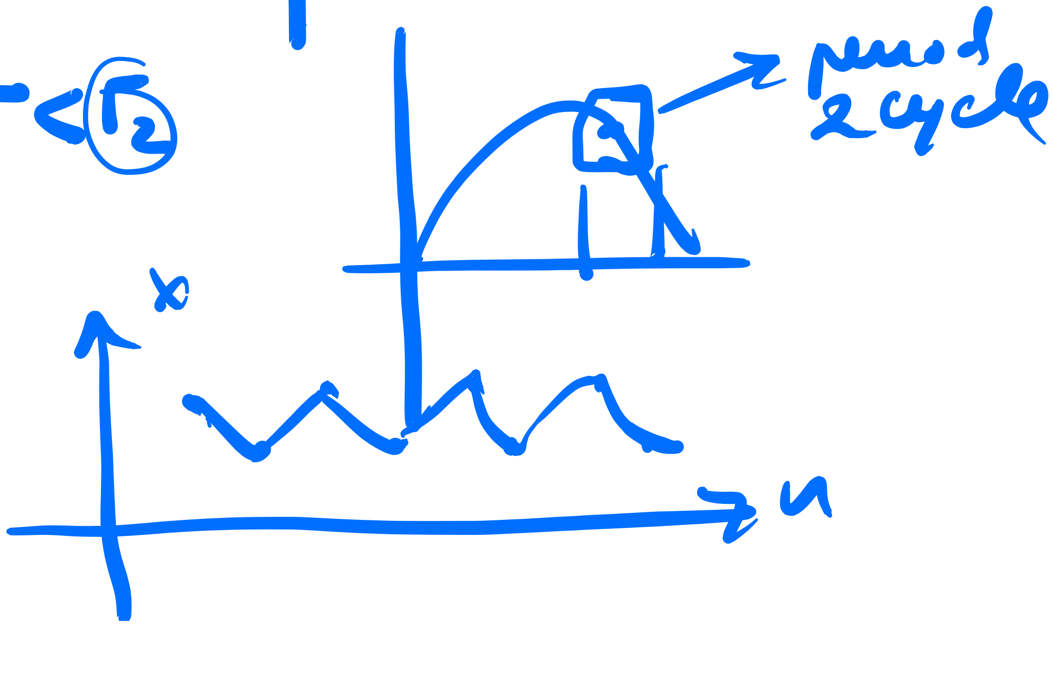
$(r-1)$



$r = 3$



$3 < r < r_2$



$r_2 \sim 3.449$

$r_2 < r < r_3$

$r_3 = 3.544$

period 8 for $r > r_3$

$r_1 < r_2 < r_3 \, \cdots \, r_\infty$

$r_n \to r_\infty$

for n large $\quad r_n \sim r_\infty - c\,\delta^{-n}$

$\delta \sim 4.x$