Manta ray attractor in a simple model for detonation waves

Rodolfo R. Rosales (MIT, Dept. of Mathematics)

Based on work with:

L. M. Faria (INRIA) and A. R. Kasimov (Skoltech)

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Abstract

Detonation waves exhibit complex dynamics, with longitudinal and transversal instabilities. Much of our understanding relies on extensive and costly numerical simulations of the reactive compressible Euler equations. Simplified theories are needed to better understand the physical processes involved. The first attempt at a reduced description was by Fickett (1979), who proposed a qualitative model. Others followed. However, these earlier models lack important physical details, and are "too stable". This lead to the statement (Joulin and Vidal, 1998):

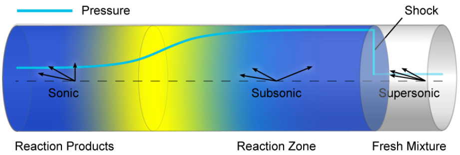
The phenomenon of detonation structures belongs to the "no theory" category because it might not be reducible to less than the compressible reactive Euler equations.

Recently improved models (yet still simple) have been proposed. The new models reproduce the complex dynamics, and can be justified asymptotically. Here I will describe one of these models, concentrating on its 1-D version, so as to describe the sequence of period doubling bifurcations associated with "galloping" detonations, as well as the resulting strange attractor; the "manta ray".

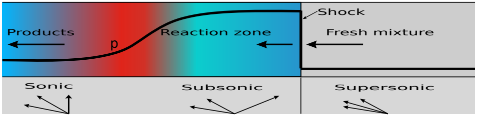
Overview

- Background and observations.
- Simplest model: scalar 1D. What it does.
- The 2D simplest model. Introduction and motivation.
- Stability analysis: numerical results.
- Nonlinear 2D dynamics (numerical). Cellular detonations.

Detonation wave structure



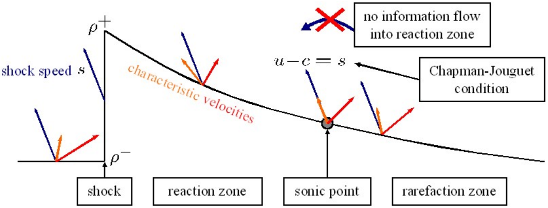
Shock waves with attached reaction, which drives the shock.



Self-sustaining detonations (Chapman-Jouguet) have a *sonic point*, insulating them from the flow behind — sound waves from behind cannot reach it.

Notation: u, c, s = flow, sound, and shock speeds; p = pressure; and ρ = density. The arrows indicate the left/right sound waves (speeds $u \pm c$) and particle paths (speed u).
DW's have complex dynamics, with many instabilities.

ZND theory# for Chapman Jouguet (CJ) detonations



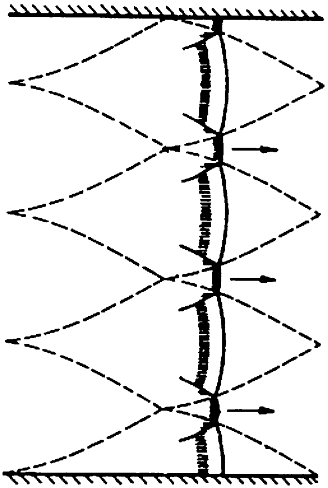
Reaction zone travels with shock. Sonic point is an event horizon — key to self-sustaining — the CJ condition $s = u - c$ determines the CJ speed. Over-compressed detonations: faster than CJ; no sonic point.

ZND dets. are planar/steady. Real detonations rarely so (ZND wave is generally unstable); exhibit regular patterns, as well as chaotic dynamics.

Cellular and spinning detonations. Transversal instability.

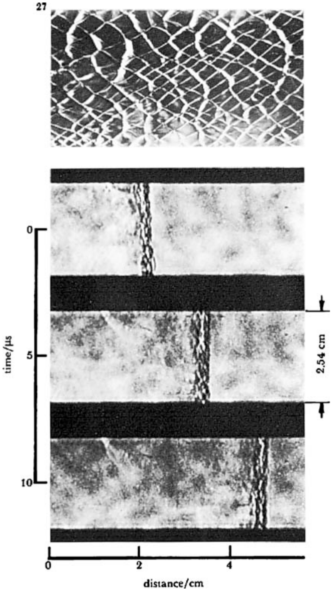
Pulsating or "galloping" detonations. Longitudinal instability.

Zeldovich, von Neumann, and Doering — during WW II.



Schematic diagram: self-sustained detonation front displaying cellular structures formed by triple point collisions.

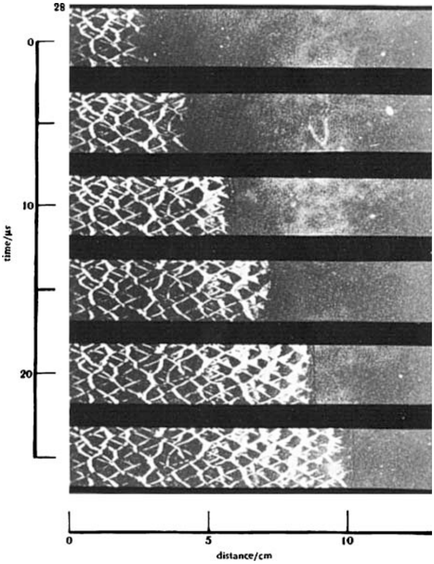
Dynamics of Combustion Systems, 2nd ed.,
A. K. Oppenheim,
Springer Verlag, 1965.
Fig 12.29 p 329.



Observations

Imprint of a self-sustained detonation front on a soot covered wall, and its cinematographic schilieren record.

Dynamics of Combustion Systems, 2nd ed., A. K. Oppenheim, Springer Verlag, 1965. Fig 12.23 p 323.



Observations

Cinematographic schlieren record of a self-sustained detonation front with its simultaneously recorded imprint on a soot covered wall.

Dynamics of Combustion Systems, 2nd ed., A. K. Oppenheim, Springer Verlag, 1965. Fig 12.24 p 324.

Simplest model: scalar 1D

Model: Fluid by inviscid Burgers' eqn. (min. needed for shocks). Reaction by source term with shock
strength dependent position: $\mathbf{u_t + (u - D)\, u_x = f(x,\, u_s)}$ for $\mathbf{x < 0}$.
Here $\mathbf{u = 0}$ for $\mathbf{x > 0}$, there is
a shock at $\mathbf{x = 0}$ (shock attached coordinates) with "lab" frame speed $\mathbf{D = \frac{1}{2}\, u_s}$, $\mathbf{u_s = u_s(t) = u(0,\, t)}$, & the heat release $\mathbf{f}$ is a function of $\mathbf{u_s}$.

Example:[1] $\mathbf{f = \frac{1}{16\sqrt{\pi\,\beta}}\, \exp\left(-\frac{\xi^2}{4\,\beta}\right)}$, $\mathbf{\xi = x - x_f}$, $\mathbf{x_f = -(2\,D)^{-\alpha}}$,

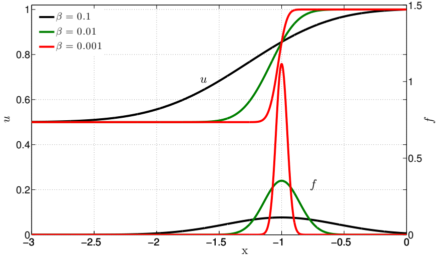
where: $0 < \beta =$ reaction zone width,
and $0 < \alpha =$ activation energy (reaction's sensitivity to shock strength).
These are the key physical effects controlling longitudinal instabilities.
Model develops pulsating detonations — from periodic to chaotic,
via a sequence of period doubling bifurcations.
Model can be justified asymptotically.

[1] **In this example the heat release profile is a fixed shape centered at $\mathbf{x_f}$.**

Scalar 1D model Chapman Jouguet (CJ) steady state

Recall model: $u_t + (u - D)\, u_x = f(x - x_f)$ for $x < 0$,

where $D = \frac{1}{2}\, u(0,\ t)$, $f(\xi) = \frac{1}{16\sqrt{\pi\,\beta}} \exp\left(-\frac{\xi^2}{4\,\beta}\right)$, and $x_f = -(2\,D)^{-\alpha}$.



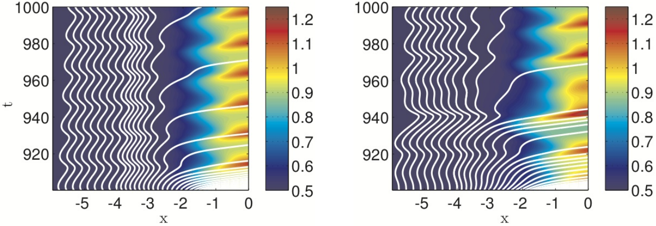
Steady state solution, and heat release function, for various β's. Given β there is an α_c such that this solution is stable for $\alpha < \alpha_c$. At α_c a **Hopf bifurcation** of the steady state occurs, followed by period doubling bifurcations.

Numerical experiments: keep $\beta = 0.1$ ($\alpha_c \approx 4.04$) fixed and increase α. The Feigenbaum constant for the period doubling is $\delta \approx 4.669$ (same as the logistic map). A chaotic attractor occurs for $\alpha \approx 5.1$.

Similar to behavior obtained via calculations with the full reacting Euler equations [Henrick-Aslam-Powers JCP2006].

Scalar 1D model period doubling

Model: $u_t + (u - D)\, u_x = f(x - x_f)$ for $x < 0$, $x_f = -(2\,D)^{-\alpha}$.



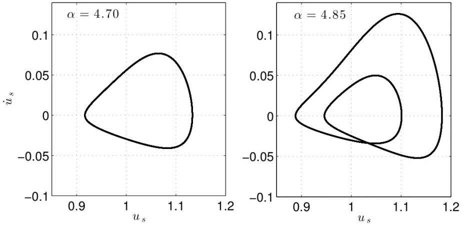
$\beta = 0.1$ for both.

Left to right:

$\alpha = 4.7$ periodic,

$\alpha = 5.1$ chaotic.

Above: solution $u(x, t)$ and characteristics $\dot{x} = u - D$ (in white).

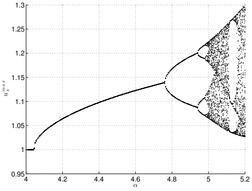


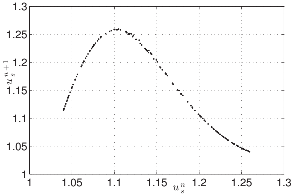
Period "one" and period "two" limit cycles in the $(\mathbf{D}, \dot{\mathbf{D}})$ plane, for $\boldsymbol{\beta} = \mathbf{0.1}$ and $\boldsymbol{\alpha} = \mathbf{4.7,\ 4.85}$.

Recall: $\mathbf{D} =$ shock velocity.

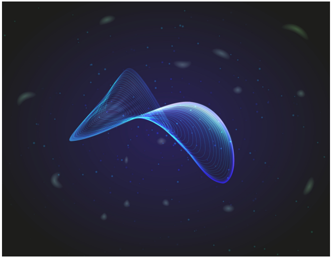
Scalar 1D model chaos and attractor

Model: $u_t + (u - D)\, u_x = f(x - x_f)$ for $x < 0$, $x_f = -(2\,D)^{-\alpha}$.





Left: local max.'s of shock strength u_s, versus α.
Right: Lorenz map: shock strength of consecutive local max.'s for $\alpha = 5.1$.



Chaotic attractor, $\alpha = 5.1$, projected on $(\mathbf{D}, \dot{\mathbf{D}}, \ddot{\mathbf{D}})$ "space". Recall $\mathbf{D} =$ shock velocity.

In this page $\beta = 0.1$.

Rendering of attractor by Olga Kasimov.

The 2D model: Introduction and motivation

Simplest 2D extension of $u_t + \frac{1}{2}(u^2)_x = 0$ (canonical eqn. for weakly nonlinear 1D shocks) is the canonical eqn. for weakly nonlinear quasi planar shocks [2]

$$\mathbf{u_t + \frac{1}{2}(u^2)_x + v_y = 0} \quad \text{and} \quad \mathbf{v_x - u_y = 0},$$

where v = transversal velocity. Thus the model [3]

$$\mathbf{u_t + \frac{1}{2}(u^2)_x + v_y = f(x - x_s, u_s)} \quad \text{and} \quad \mathbf{v_x - u_y = 0} \quad \text{for} \quad \mathbf{x < x_s},$$

where $x_s = x_s(y,\, t) =$ shock position, $u_s = u_s(y,\, t) =$ u-value behind the shock, $f =$ heat release function, and $u = v = 0$ for $x > x_s$. At the shock:

$$\mathbf{(x_s)_t\, u_s - \frac{1}{2}\, u_s^2 + (x_s)_y\, v_s = 0} \quad \text{and} \quad \mathbf{(x_s)_y\, u_s + v_s = 0},$$

where $v_s =$ value of v right behind shock, and $\mathbf{u_s > 0}$ (entropy condition).

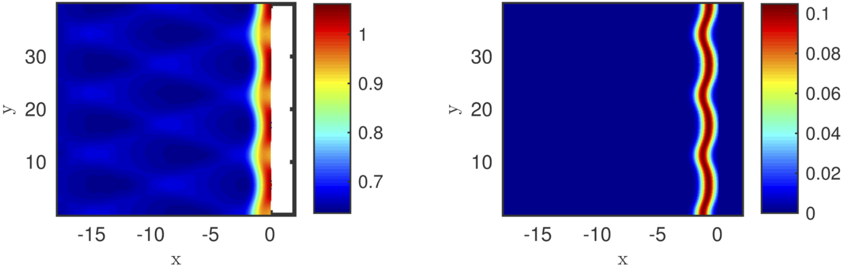
These eqns. have plane steady solutions (ZND detonations).

[2] Small disturbance transonic flow [Lin-Reissner-Tsien 1947], nonlinear acoustics [Zabolotskaya-Khokhlov 1969], etc.

[3] Extension to 3D: use potential form of the equations. **Note: we no longer use shock-attached coordinates.**

Nonlinear 2D dynamics (numerical): Cellular detonations

Numerically we investigate the long time dynamics of the instabilities, starting from a slightly perturbed (unstable) ZND detonation. When the stability is weak, a very regular multi-D cellular pattern emerges, with smooth transverse waves:

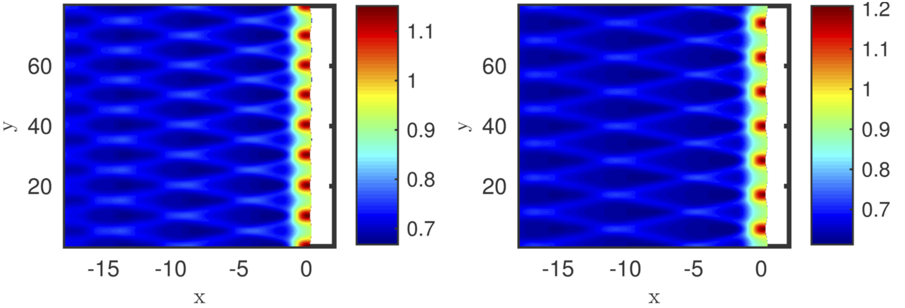


Large time dynamics ($t = 800$) for $\beta = 0.1$, $\alpha = 3.5$, $\zeta = 1.05$ (left u, right f). As the instability grows the transverse waves get sharper, the structure becomes more complex, and eventually (possibly) chaotic.[4] We illustrate this next.

[4] **We have not studied the various bifurcations that seem to be involved.**

Cellular detonations (vary overdrive ζ = speed/CJ-speed)

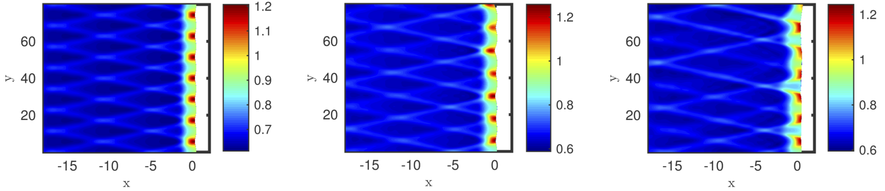
Waves closer to the C-J case ($\zeta = 1$) are more unstable and display stronger transverse variations. Furthermore, for smaller overdrive the cells become larger (consistent with linear stability).

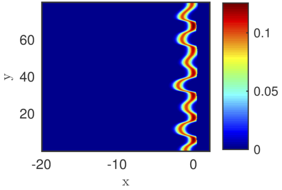


Plots of u at $t = 1000$ showing cellular patterns for $\alpha = 4.05$ and $\beta = 0.1$. Here $\zeta = 1.10$ on the left, and $\zeta = 1.05$ on the right.

Cellular detonations (vary activation energy α)

Here we explore the reaction shock-state sensitivity effect on the slns. As α grows, so does the plane waves instability $\Rightarrow$ increasingly complex cellular patterns.





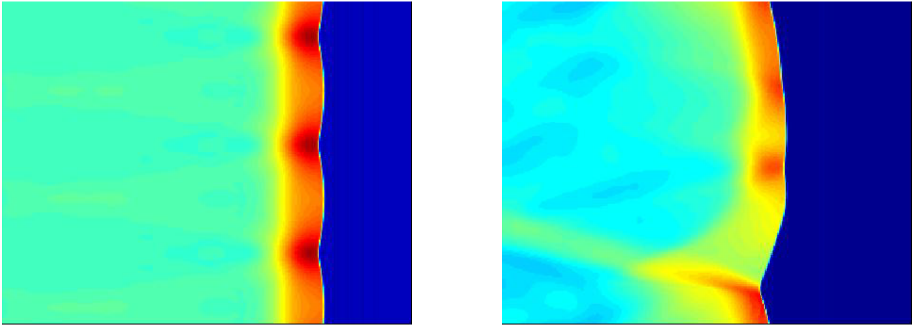
Large time ($t = 1000$) behavior: $\zeta = 1.05$, $\beta = 0.1$, and $\alpha = 4.05, 4.30, 4.50$ (top, left to right). Bottom: corresponding f for $\alpha = 4.50$.
The increasing complexity of the pattern is evident as α grows, with a fairly irregular behavior for $\alpha = 4.50$.

A quantitative characterization of the 2D dynamics, of the sort shown earlier for the 1D model, is challenging. It does seem, however, that the solutions do go through a series of bifurcations as α grows.

Simplest 2D model: Transversal Instabilities (movies)

Eqns.: $\mathbf{u_t + (\frac{1}{2}\,u^2)_x + v_y = f(x - \sigma - x_f)}$, and $\mathbf{v_x = u_y}$,

where $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{t},\,\mathbf{y})$ = shock position, $\mathbf{D} = \boldsymbol{\sigma}_\mathbf{t}$ = shock speed, $\mathbf{u = v = 0}$ for $\mathbf{x} > \boldsymbol{\sigma}$, etc. In particular $\mathbf{f(\xi) = \frac{1}{8\sqrt{\pi\,\beta}}\,\exp\left(-\frac{\xi^2}{4\,\beta}\right)}$ and $\mathbf{x_f = -(2\,D)^{-\alpha}}$.



Examples of transversal instabilities, $\alpha = 3.9$ (left) and $\alpha = 4.5$ (right) $\beta = 0.1$.

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