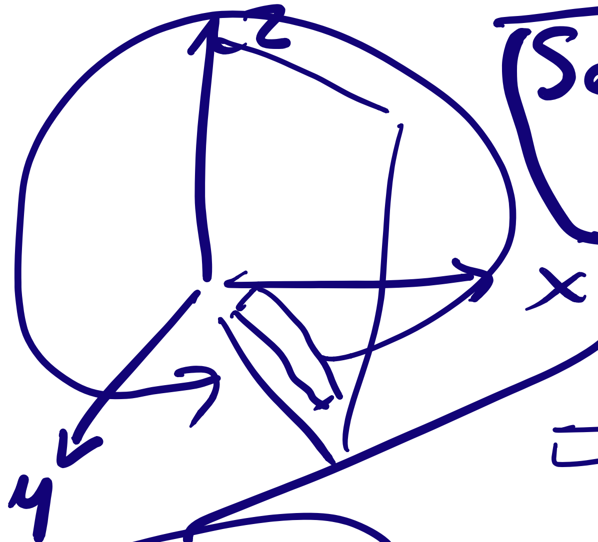
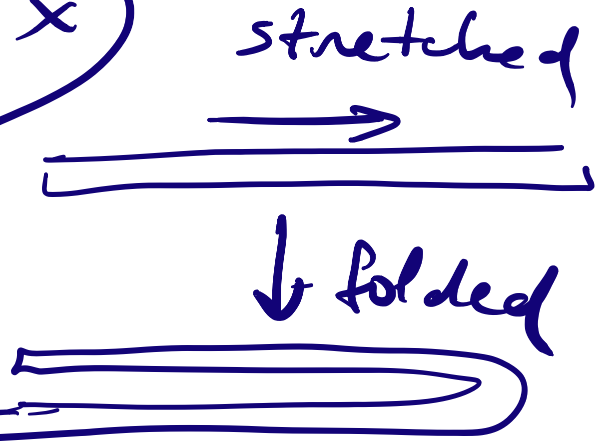
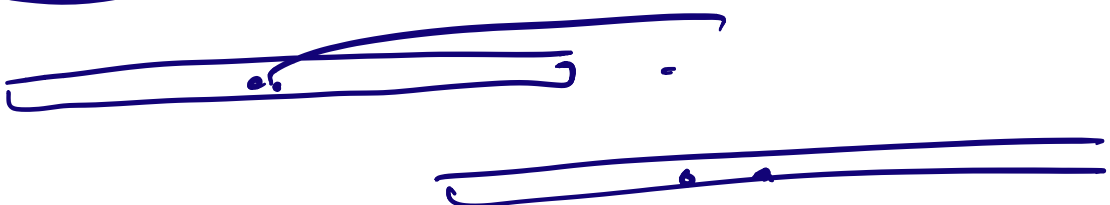


Sensitive dependence on I.C.

+ Butterfly Effect





distance between orbit

$$\sim e^{\lambda t}, \quad \lambda > 0$$

λ Floquet exponent

ϵ = error in the I.C.

δ = desired tolerance in answer after time T

$$\epsilon e^{\lambda T} \le \delta$$

Generally

$\frac{\epsilon}{\delta}$ is small when λ is large

$$T \le \frac{1}{\lambda} \ln(\delta/\epsilon)$$

$$\boxed{T_{max} = \frac{1}{\lambda} \ln\left(\frac{\delta}{\epsilon}\right)}$$

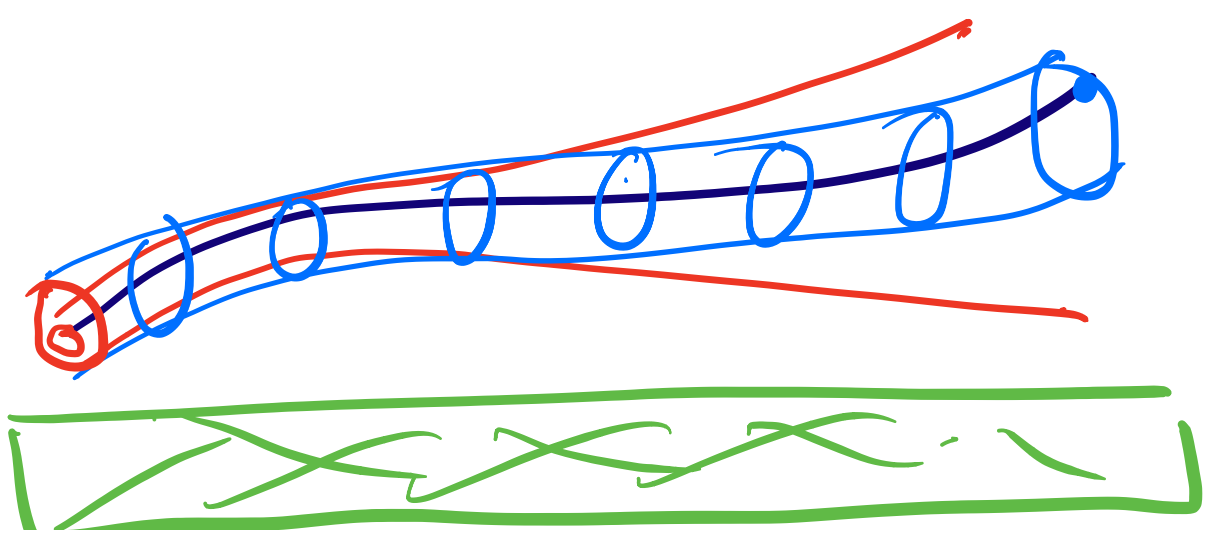
If you want T longer then need to reduce ϵ!

$$T_m \to n T_m$$

$$n T_m = \frac{1}{\lambda} \ln\left(\frac{\delta}{\epsilon_n}\right)$$

$$\frac{1}{\lambda} \log\left(\frac{\delta}{\epsilon}\right)$$

$$\Rightarrow \epsilon_n = \delta \left(\frac{\epsilon}{\delta}\right)^n$$



Takens' embedding theorem

observable

$$f(t)$$

$$\vec{x}(t) \in P$$

$$f(t) = F(x(t))$$

$$\vec{v} = [f(t), f(t-T), f(t-2T), \dots f(t-(n-1)T)] \in R^n$$

$$\boxed{T > 0}$$

ω_1 ω_2 ω_3 ω_3

$(\omega_1, \omega_2, \omega_3)$ $(\omega_2, \omega_3, \omega_4)$ - -

$$\boxed{\vec{V} = [\omega_\ell, \omega_{\ell-1} \, - - \, \omega_{\ell-n+1}]}$$

$$[f(t), \dot{f}(t), \ddot{f}(t)]$$

Manta Ray Attractor

"Detonation wave"

$$\rho_t + (\rho u)_x = 0$$

$$(\rho u)_t + (\rho u^2 + p)_x = \varpi$$

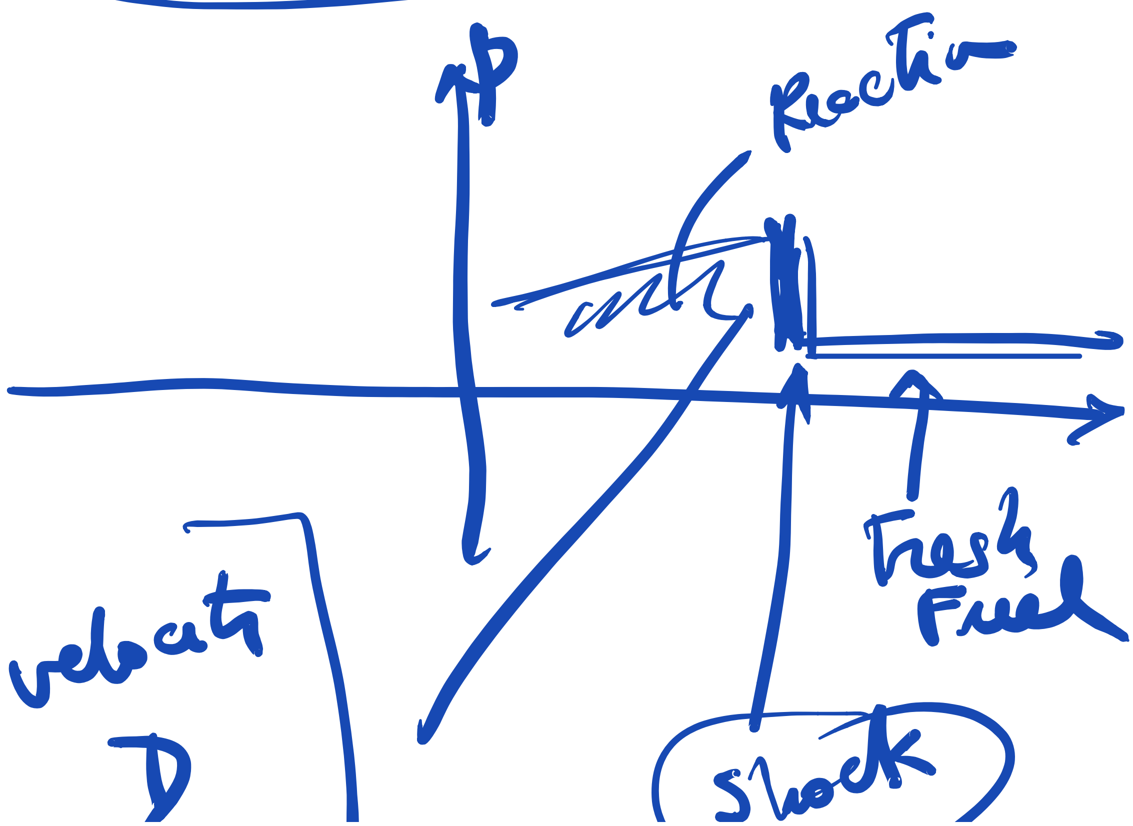
$$(\rho E)_t + (\rho u E + up)_x = Wu$$

$$E = \frac{1}{2}u^2 + e \quad , \quad \underline{e = e(T)}$$

$$e = e(p,\rho) = \left(\frac{p}{\rho}\right)^{\gamma-1} \quad (?)$$

$$\underline{W = W(T, p, \rho, Z)}$$

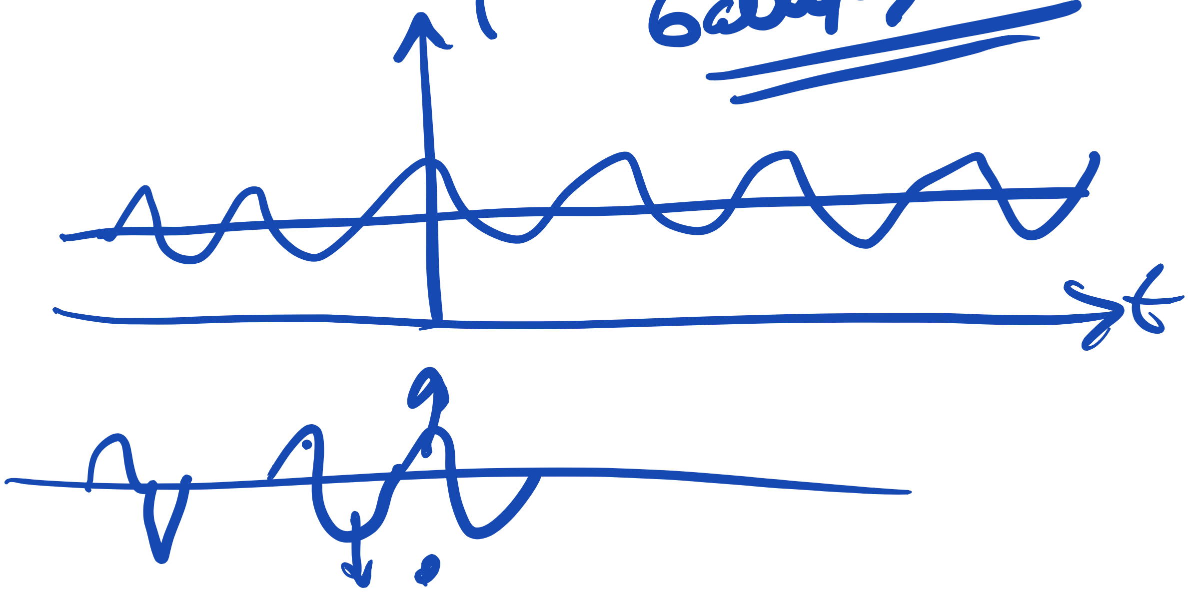
$$\boxed{Z_t + u Z_x = -R}$$

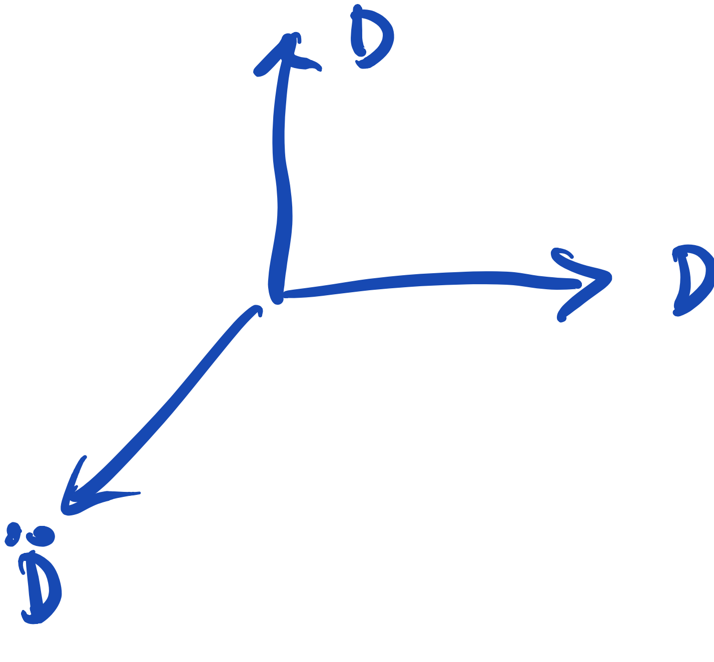


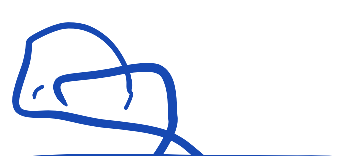
count

$D = D(t)$

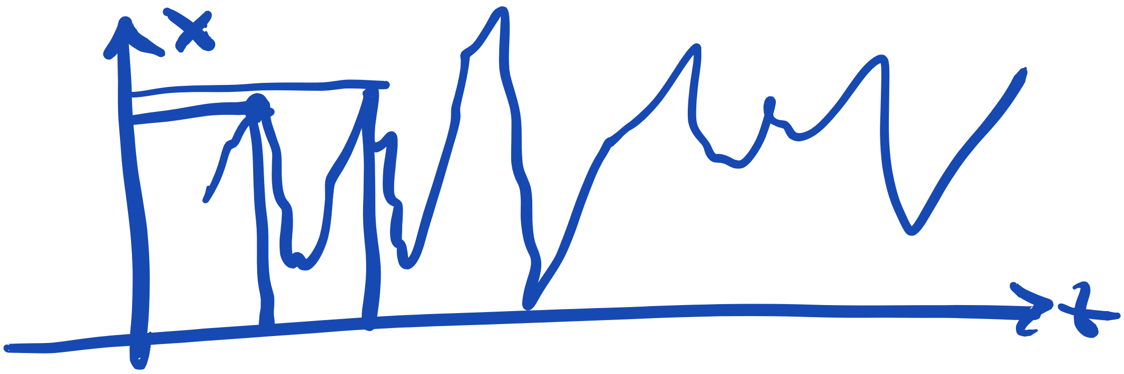
Galloping dot.



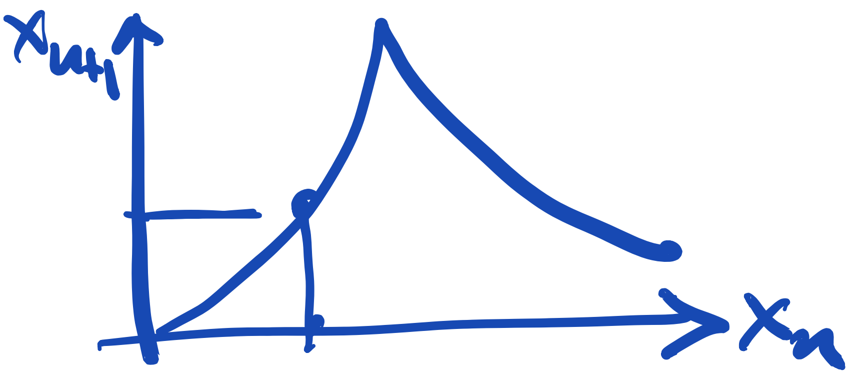




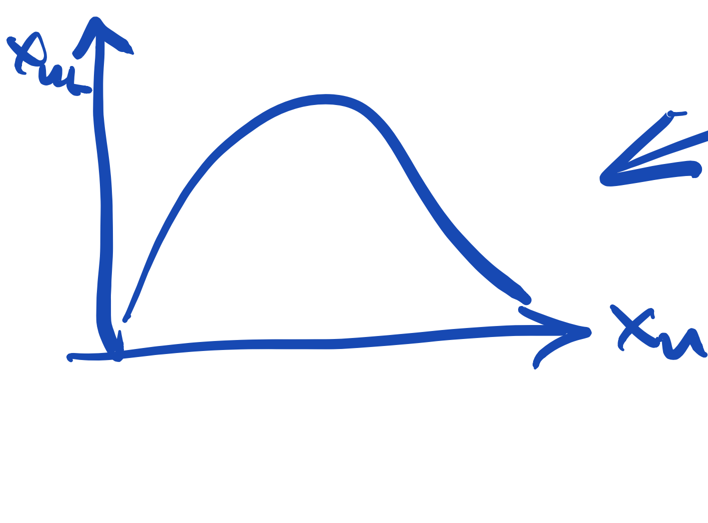
Lorenz map



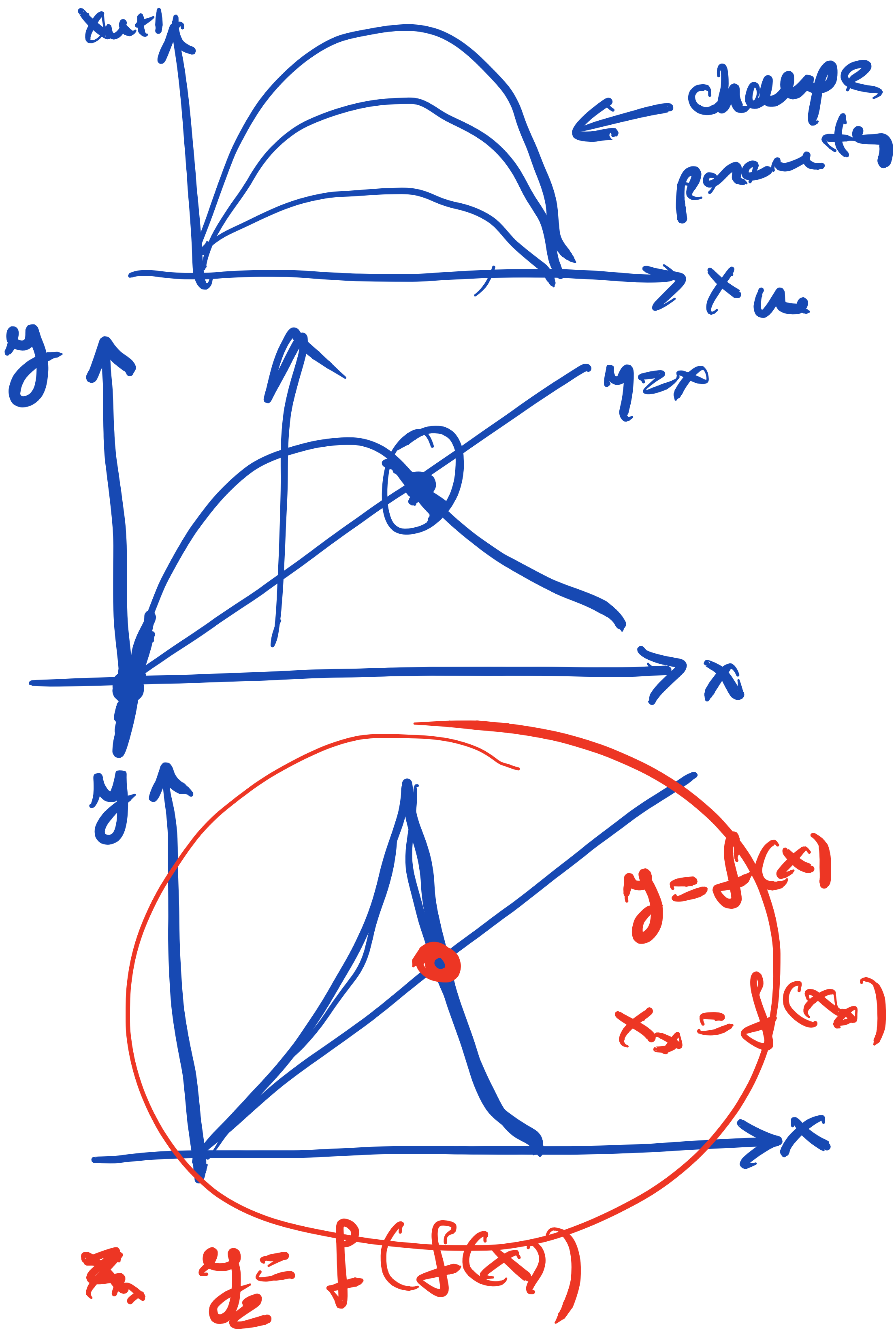
$x_n, x_{n+1}, x_{n+2}, \ldots$



(x_n, x_{n+1}) Lorenz



Rossler or mackey



x_{n+1}
change parameter
x_n
y
y=x
x
y
x
y = f(x)
x* = f(x*)
y_2 = f(f(x))

$$y_n = \underbrace{f(f(f \cdots f(x))}_{n \text{ times}}$$

$$y_1 \longrightarrow f'(x_0)$$

$$y_2 \longrightarrow f'(f(x_0))\, f'(x_0)$$

$$y_3 \longrightarrow \underline{f'(f(f(x_0)))}\, f'\underline{[f(x_0)]}\, f'(x_0)$$