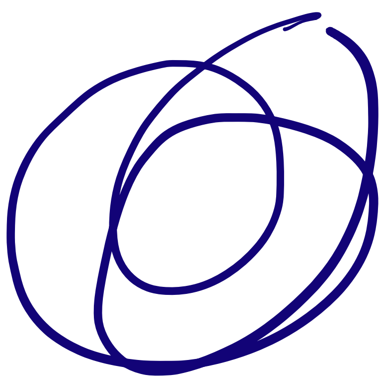
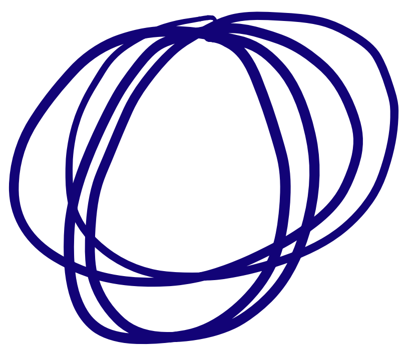
Lissajous Figs

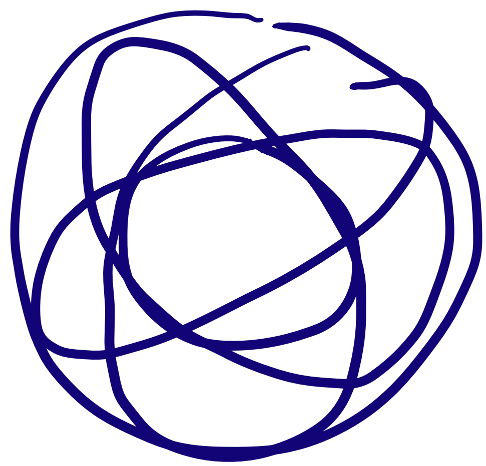
$$\begin{cases} x = \cos \omega_1 t \\ y = \sin \omega_2 t \end{cases}$$

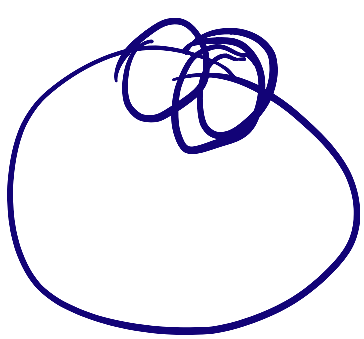
$$\frac{\omega_1}{\omega_2} = 2/3$$



P. Set







Cont. with Dimension

Self similarity

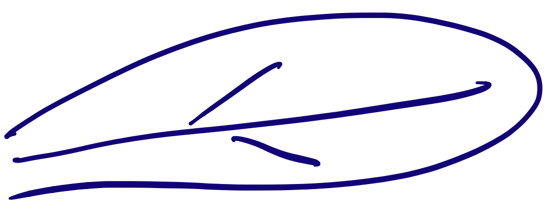
Set = N subsets of length r

with subsets = to original

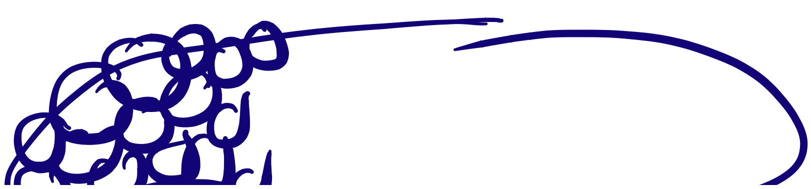
upon stretching by 1/r

$$\boxed{d = -\ln N / \log r}$$

$$N \sim r^{-d}$$



Box dimension



Covering has N balls

Let N be min needed

$r \longrightarrow N$ $\quad \boxed{N = N(r)}$

"Regular set" $\quad N \sim 1/r^2$
- in plane

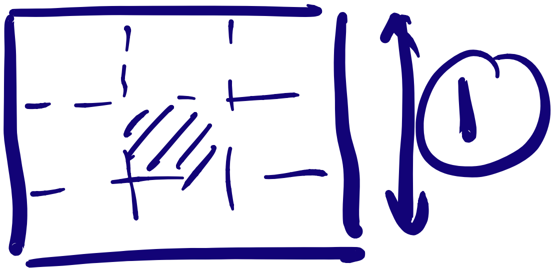
---

if $N \sim \underline{\underline{r^{-d}}}$

$$\boxed{d = -\lim_{r \to 0} \log N / \log r}$$

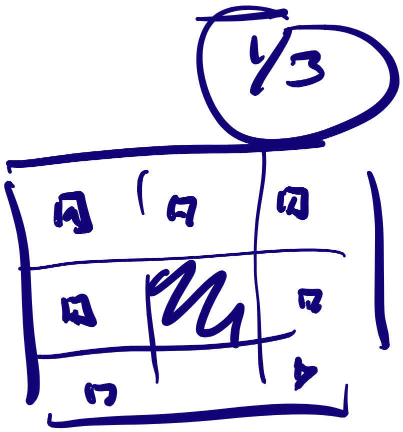
if the limit exists!

Generalizes self-similar dimension



$r = \left(\frac{1}{3}\right)^n$

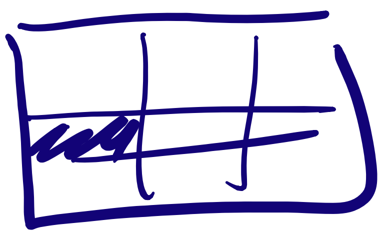
$N = 8^n$



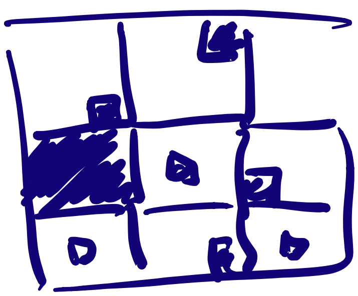
$$\boxed{d = +\frac{\ln 8}{\ln 3}}$$

Self similar

Not self similar



$r = \left(\frac{1}{3}\right)^n$



$N = 8^n$

$d = \ln 8 / \ln 3$

Example Box dimension of

1. Interval $[0,1] \longrightarrow 1$
2. Rationals in $[0,1] \longrightarrow 1$
3. Irrational in $[0,1] \longrightarrow 1$

$\boxed{\text{Hausdorff dimension}}$

$d = \dim H(X) =$

$= \inf \underline{A}$

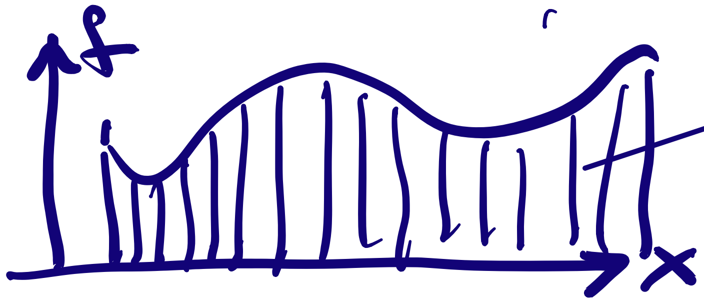
A = set of # with the property

For any $\epsilon > 0$, there is a $\delta > 0$
and a cover U of $\underline{X}$ such that

1 - All sets in U have diameters $< \delta$

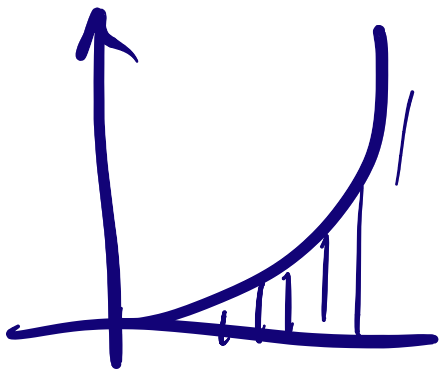
2 - $\underset{B \text{ in } U}{\text{Sum}} (\text{diam } B)^R < \epsilon$

Riemann Integral vs. Lebesgue integral

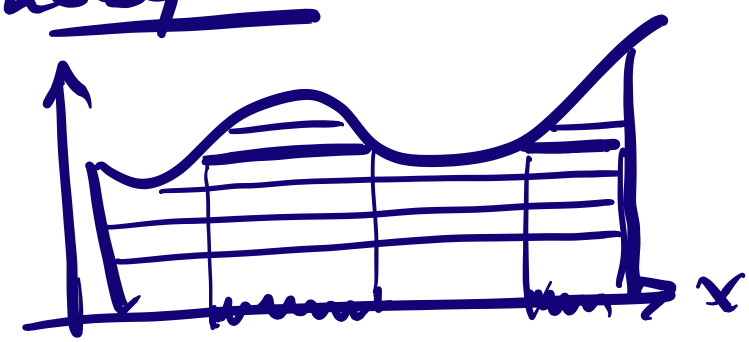


$A = ?$

lim Σ area rect.



Lebesgue



Def. of $\alpha \in A$

For any $\epsilon > 0$ there is $\delta > 0$ and a cover U of A and

1- $\mathrm{diam}(B) < \delta$ if B in U

2- $\sum \mathrm{diam}(B)^{\alpha} < \epsilon$

Suppose all sets in U have the <u>same</u> diameter r

(2) $N r^{\alpha} < \epsilon$

<u>If set</u> has box dimension d_b

<u>Then</u> $N \sim C r^{-d_b}$

<u>Take</u> $\underline{\alpha > d_b}$

$$N r^{\alpha} = (N r^{+d_b})\, r^{\alpha - d_b}$$

~~diam~~

$d_b + \epsilon$ is in A

$\therefore$ ~~Et~~ $\operatorname{Inf} A \le d_b$

Example $\boxed{d_H([0\,1]) = 1}$

Proof if $a<1$ and $\delta <= 1$

$$\sum_B (\operatorname{diam} B)^a > \sum_B (\operatorname{diam} B) \ge 1$$

Example $d_H[[0,1] \cap \mathbb{Q}] = 0$

Proof Order the rational #

$q_1, q_2, \ldots$

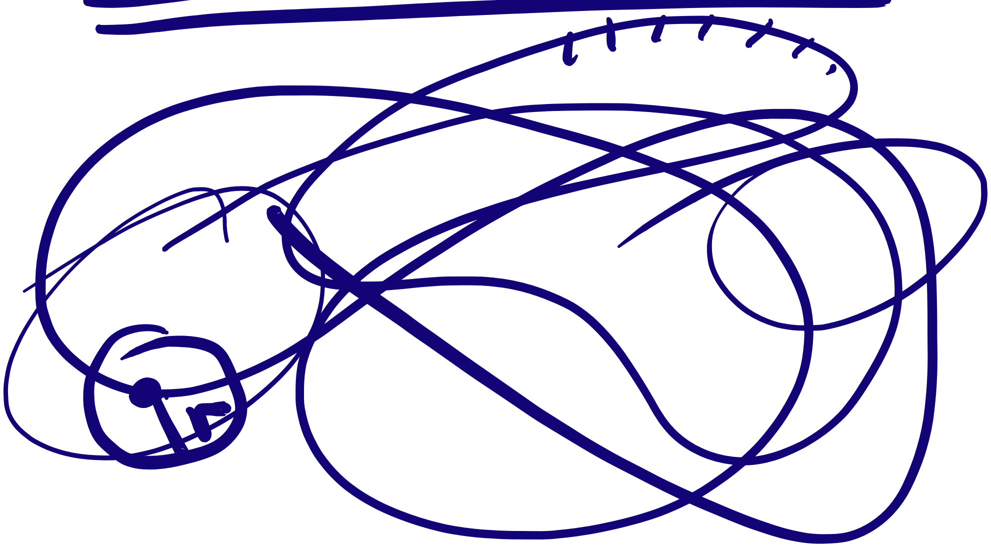
Take $0<r<1$ $U(r) =$ covers of

Set Interval of diam r^n centered at q_n

$$\sum_B (\operatorname{diam} B)^a = \sum_n r^{na}$$

$$= \frac{r^a}{1-r^a}$$

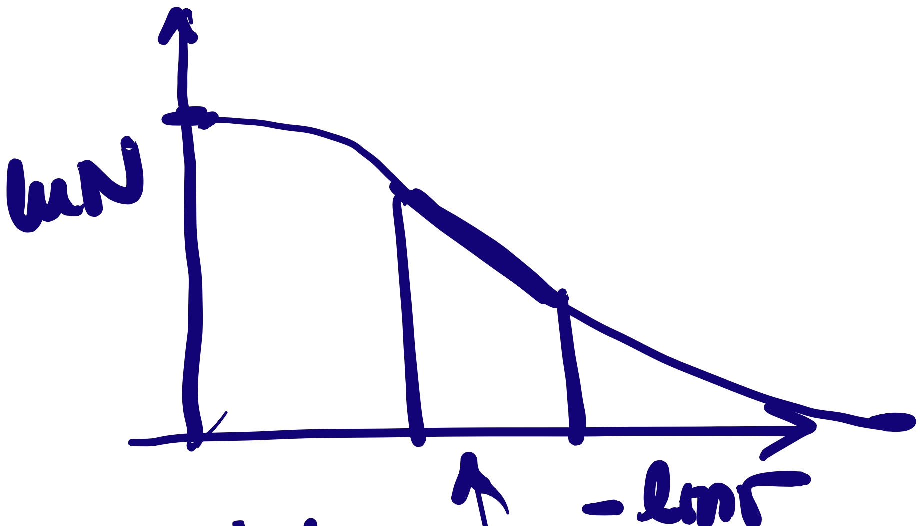
Pointwise dimension



$N(x,r)$ = # of point in target within r of x

Then if $N \sim r^d$

d = point-wise dim



$\ln N \sim d \ln r$

want this "thick"

$d = 2.05 \pm 0.15$