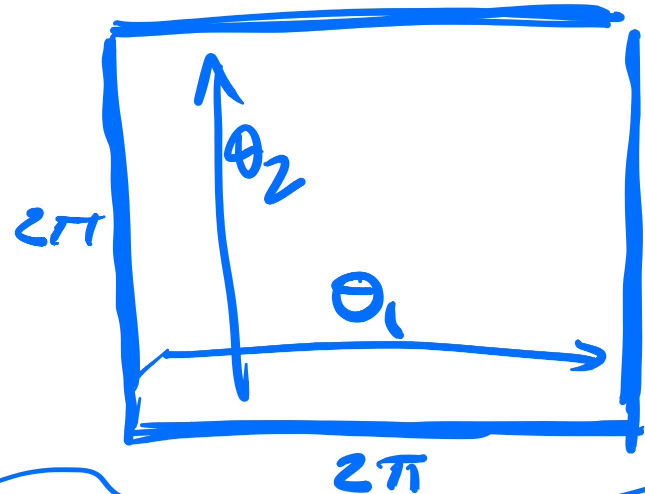
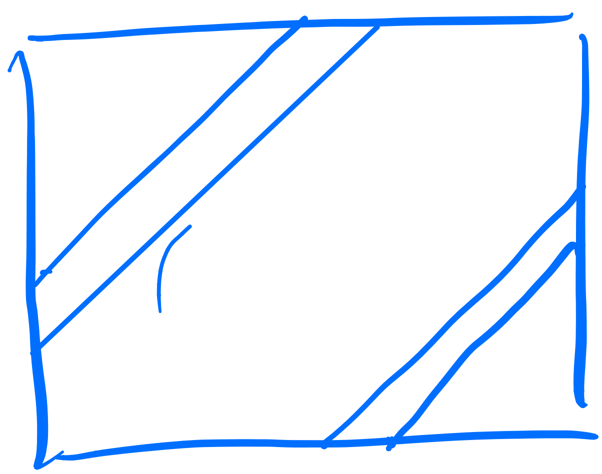
Flow on the Torus

T_2

periodic



$\dot{\theta}_1 = \omega_1 \qquad \dot{\theta}_2 = \omega_2$



$\theta_1 = \omega_1 t + \theta_{01}$

$\theta_2 = \omega_2 t + \theta_{02}$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{p}{q}}$$

$$\theta_1(t+T)-\theta_1(t)=\omega_1 T \sim 0$$

$$\theta_2(t+T)-\theta_2(t)=\omega_2 T \sim 0 \quad \underline{\bmod 2\pi}$$

$$\omega_1 T = 2\pi n \qquad n = p$$

$$\omega_2 T = 2\pi m \qquad m = q$$

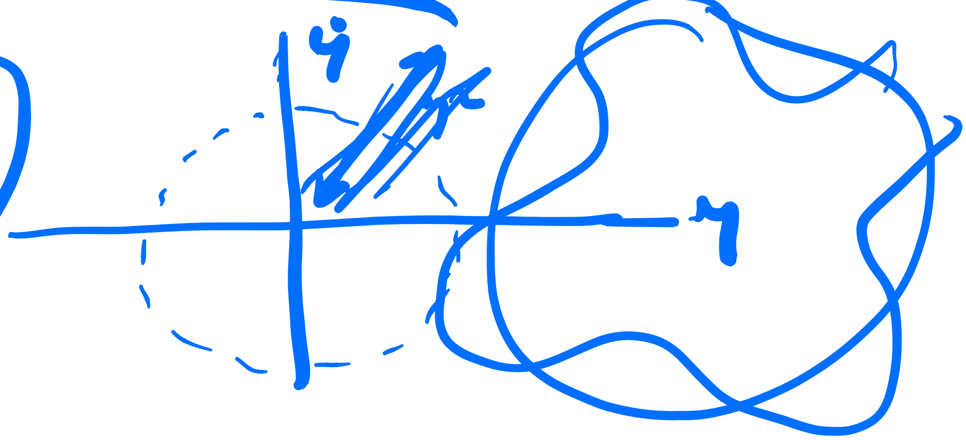
$$\boxed{T = \frac{2\pi p}{\omega_1} = \frac{2\pi q}{\omega_2}}$$

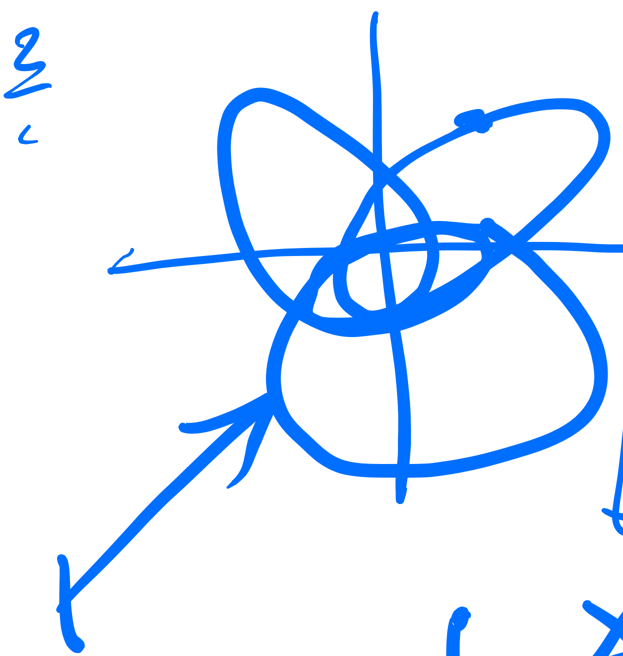
$$\ddot{y}_1 + \omega_1 y_1 = 0$$

$$\ddot{y}_2 + \omega_2 y_2 = 0$$

$$\boxed{y = \underline{y_1 + y_2}}$$

$$\frac{\omega_1}{\omega_2} = \frac{2}{3}$$





Trifoil

Lissajous Figure

$X = a \sin \omega$

$$x = a_1 \cos \omega_1 t + a_2 \cos \omega_2 t$$

$$y = a_1 \sin \omega_1 t + a_2 \sin \omega_2 t$$

Quasiperiodic Functions

Completely Integrable

Hamiltonian
Systems

$$u_{tt} - \Delta u + f(u) = 0$$

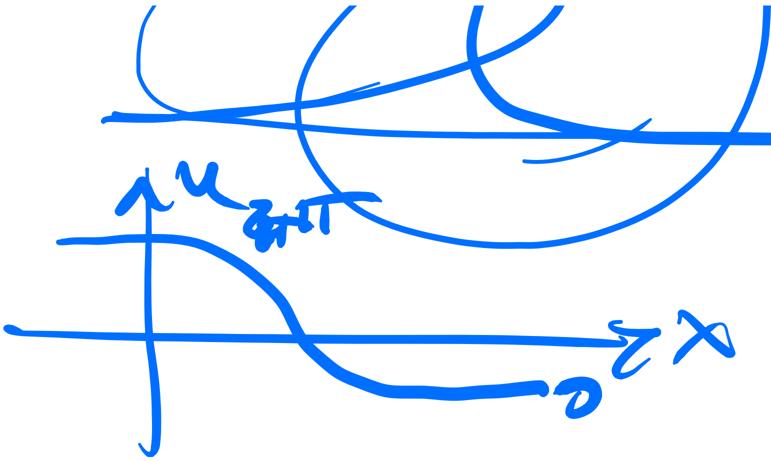
f is periodic of period 2π

$f = \alpha u$ Klein Gordon eqn.

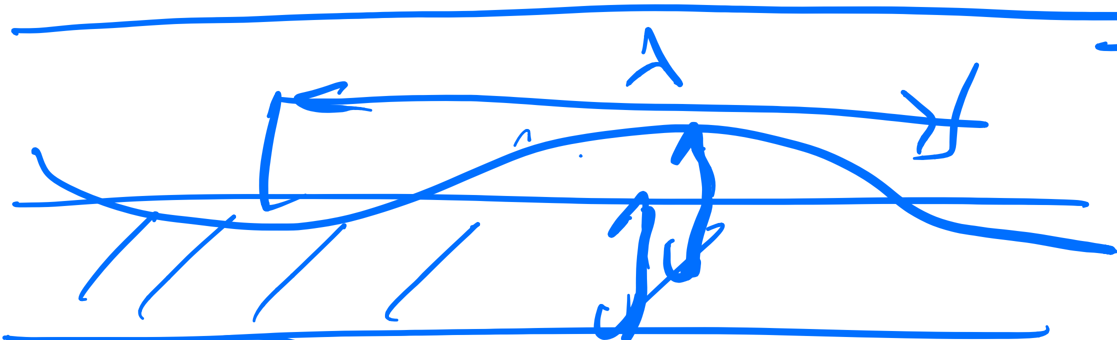
$$u_{tt} - \Delta u + \sin u = 0$$

Sine Gordon Eqn

$$u_{tt} - u_{xx} + \sin u = 0$$

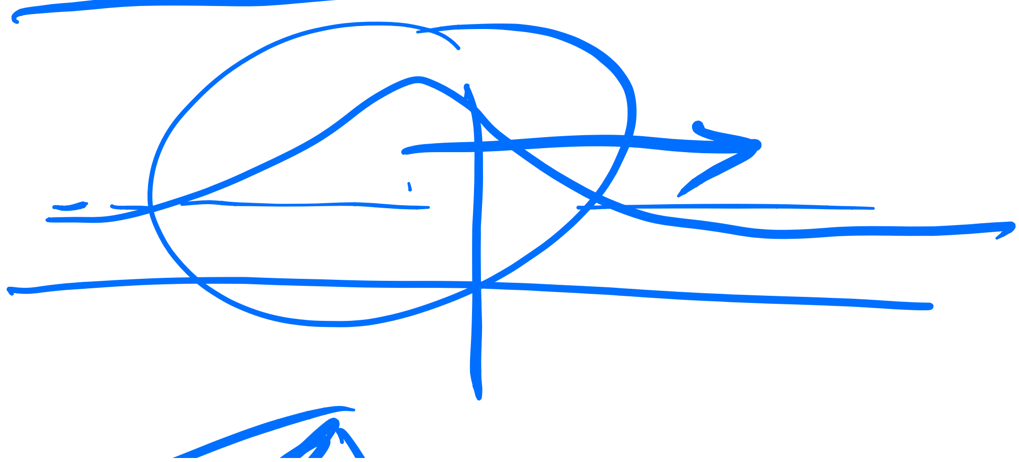


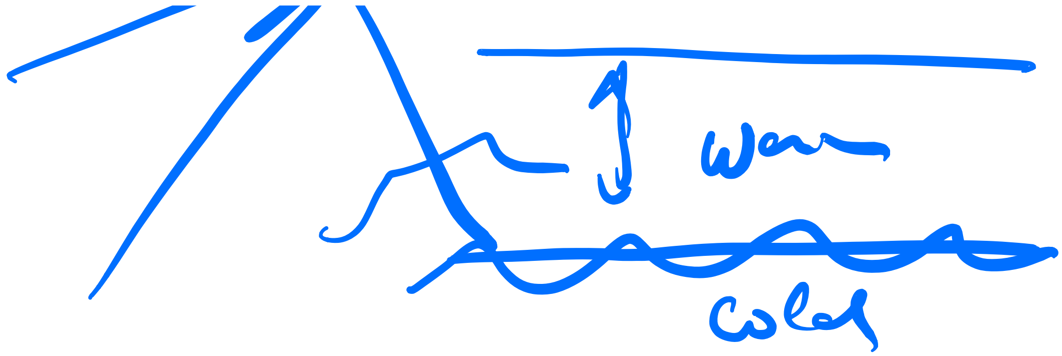
Kinks



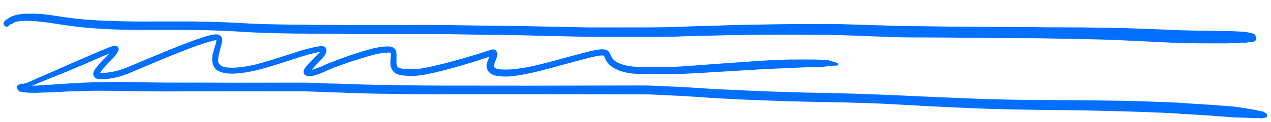
$\lambda \gg h$ as usual but interaction

$$u_t + c u_x + d u u_x + e u_{xxx} = 0$$





Solitons, instantons,
oscillons, breathers, etc



Almost Periodic

$$\boxed{y(t) = \sum a_n e^{i\omega_n t}}$$

ω_n descrete set of frequencies

Periodic $\boxed{\omega_n = n\omega}$

Quasiperiodic $\omega_n = n_1\omega_1 + n_2\omega_2$
2 period

Why are they called almost periodic

Math def

For every $\epsilon > 0$ there is one almost period $T > 0$ such that $|y(t+T) - y(t)| < \epsilon$

Landau's hypothesis

related to

Ruelle Takens

"Route to chaos"

Related to "small divisors"

$$\ddot{x}_1 + \omega_1^2 x_1 = \epsilon f(x_1, x_2)$$

$$\ddot{x}_2 + \omega_2^2 x_2 = \epsilon g(x_1, x_2)$$

$$\ddot{x} + \omega^2 x = \epsilon x^3$$

$x = \sin \omega t$

$f(x) - (\sin^3 \omega)$

$x_1 \sim \sin\omega_1 t$ $\quad$ $\omega_1 \pm \omega_2$, $2\omega_2$, $2\omega_1$

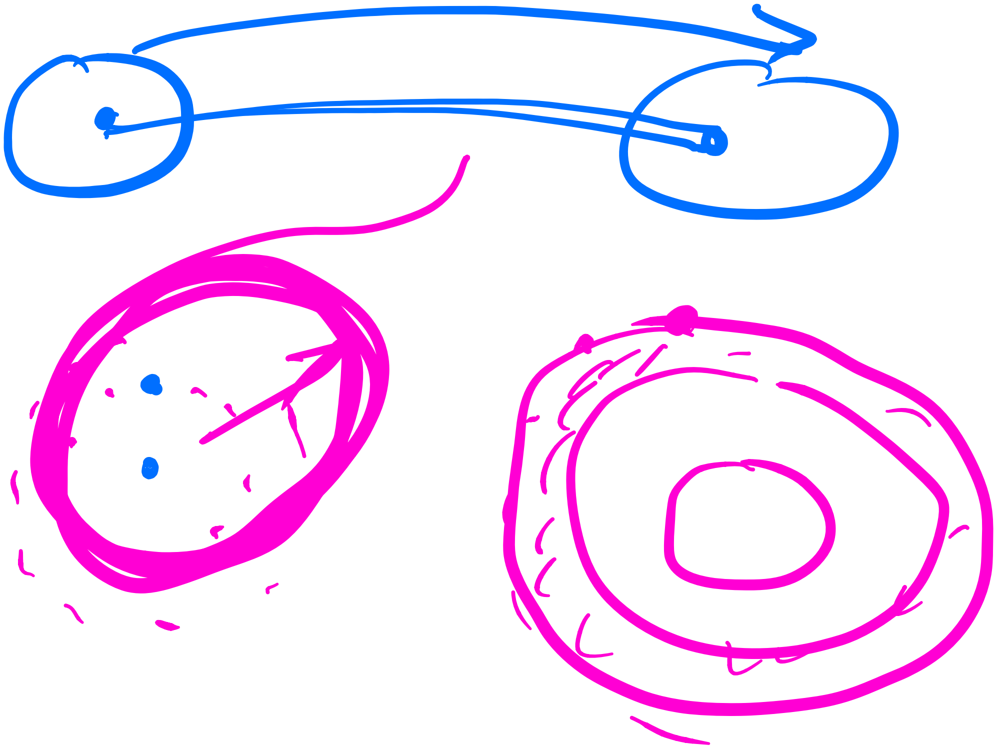
$x_2 \sim \sin\omega_2 t$ $\quad$ $\boxed{n\omega_1 + m\omega_2}$

$$\ddot{x} + \omega^2 x = e^{i\omega t}\, e^{i2\omega t} \cdots$$

$$\ddot{x} + \omega_1^2 x = e^{i(n\omega_1 + m\omega_2)t}$$

$$\boxed{x = \frac{e^{i(n\omega_1 + m\omega_2)t}}{(n\omega_1 + m\omega_2)^2 - \omega_1^2}}$$

Example of KAM



PHPC demo B