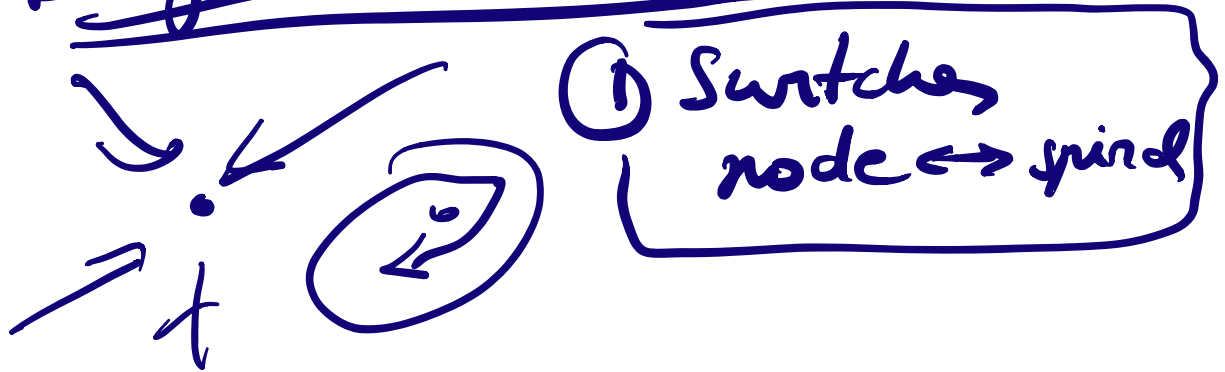


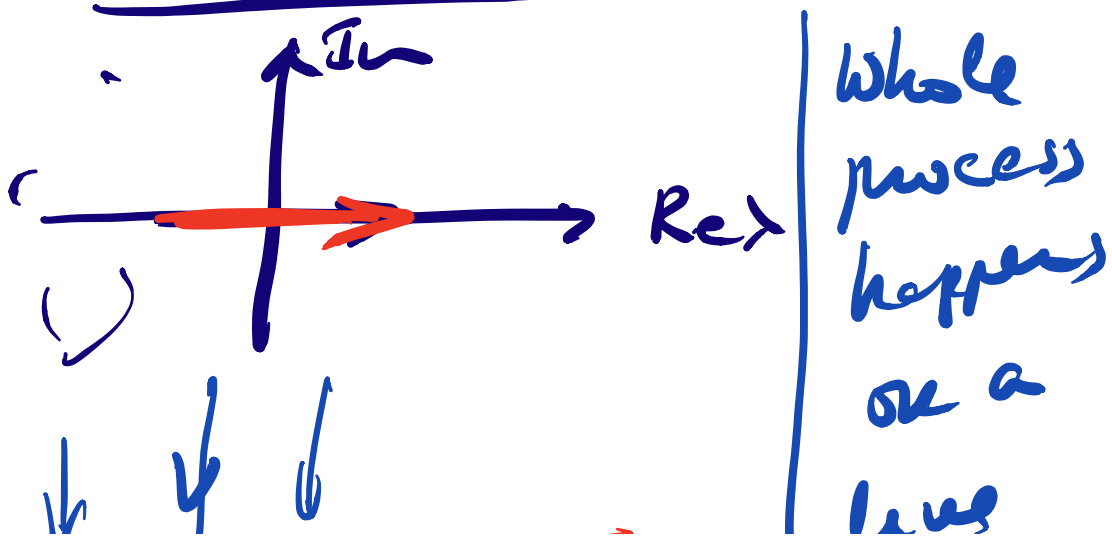
Reference in the plane



C.P. loop stability

① One eigenvalue crosses
from $\text{Re}(s) < 0$ $\text{Re}(s) > 0$

has to be real eigenvalue

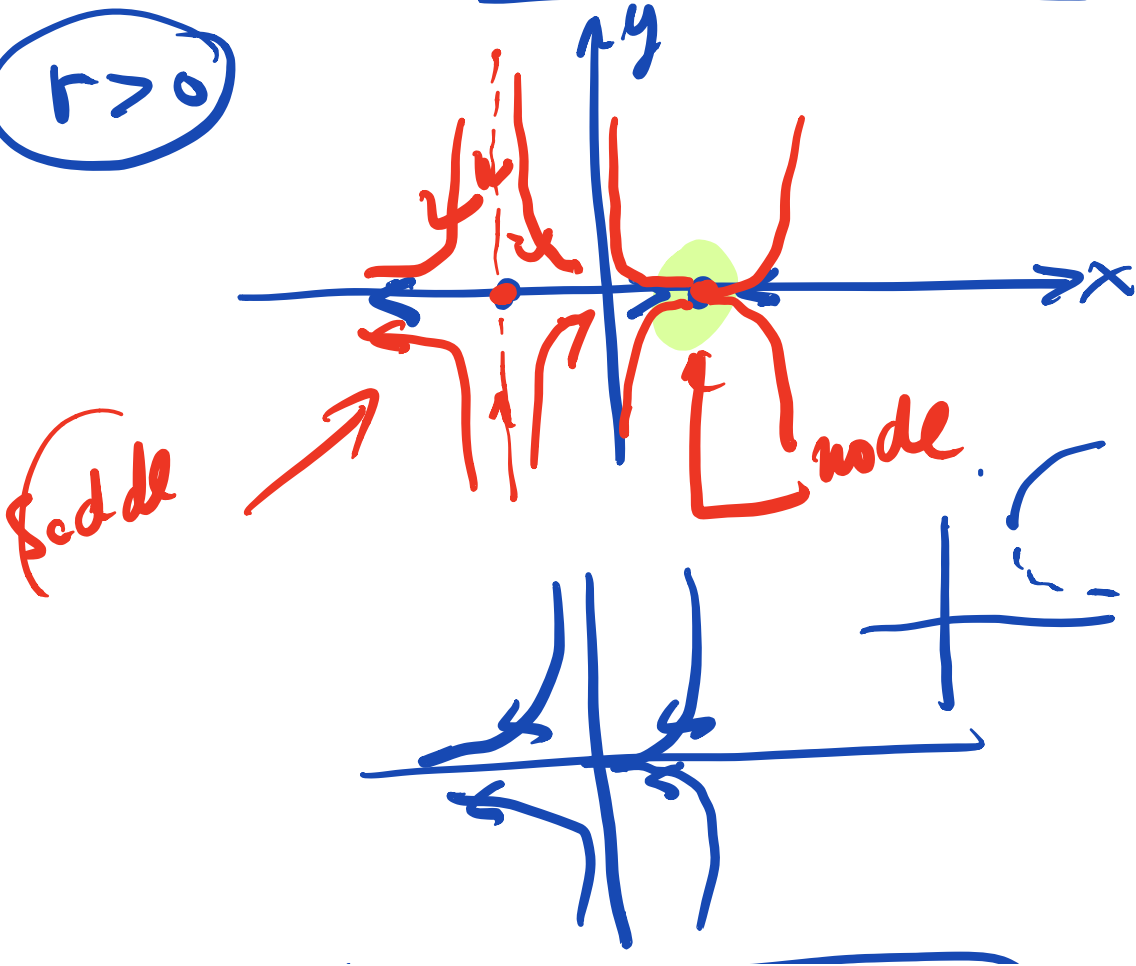




Saddle node

$$\begin{aligned} \dot{x} &= r - x^2 \\ \dot{y} &= -y \end{aligned}$$

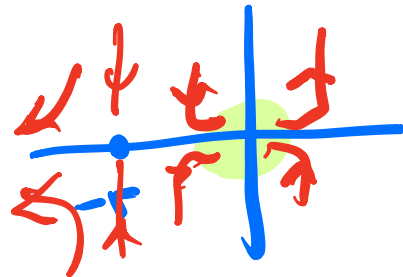
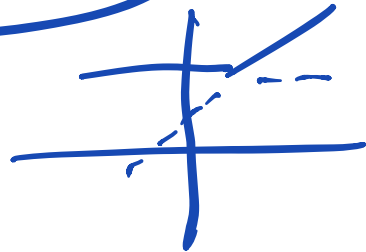
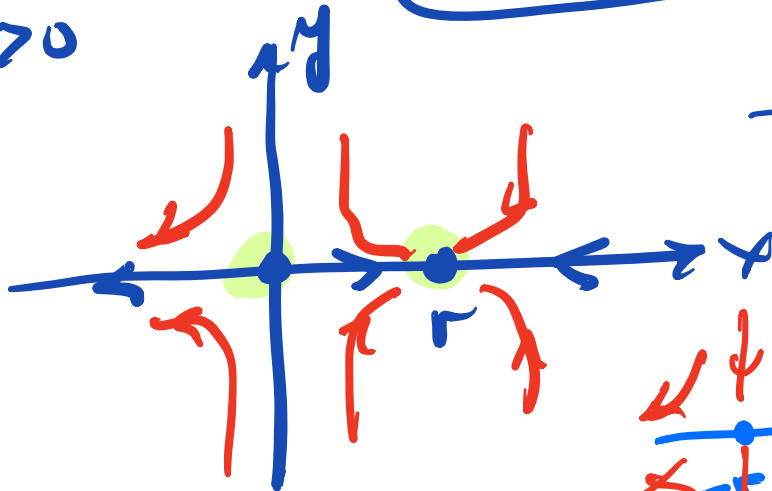
$r > 0$



Transcritical

$$\left. \begin{aligned} \dot{x} &= rx - x^2 \\ \dot{y} &= -y \end{aligned} \right\} \text{C. Form}$$

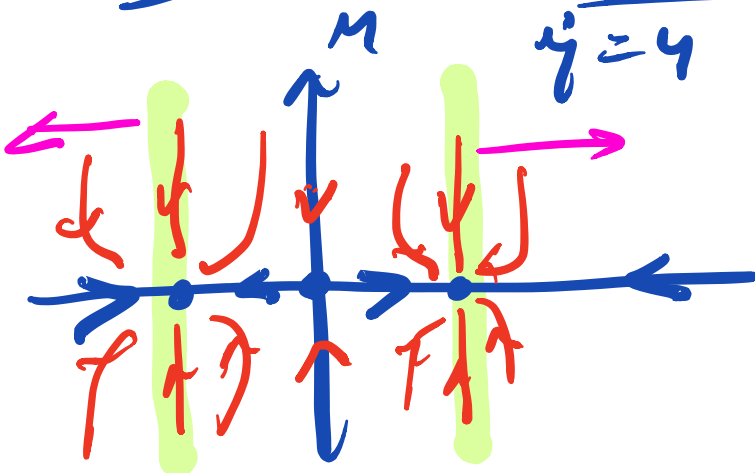
$r > 0$



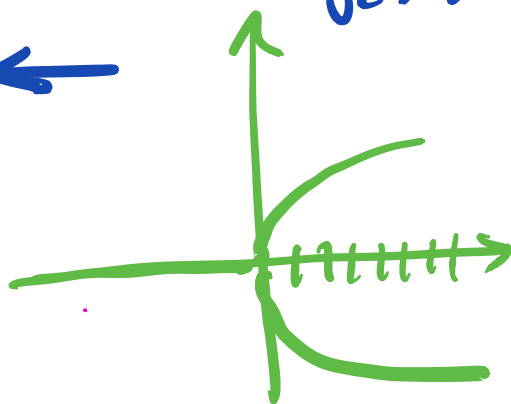
Pitchfork

$$\dot{x} = rx \pm x^3$$

$$\dot{y} = y$$

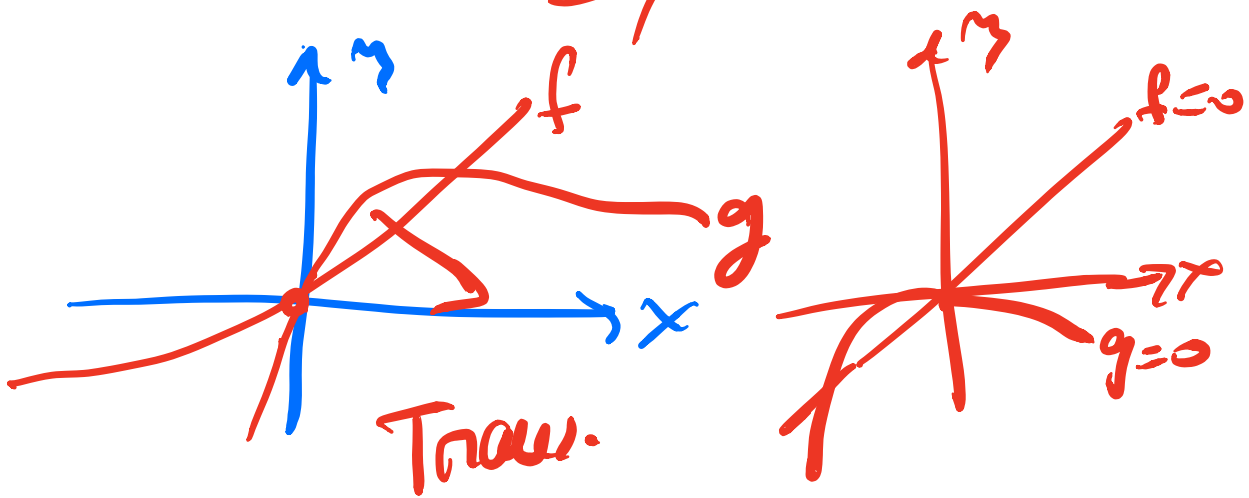
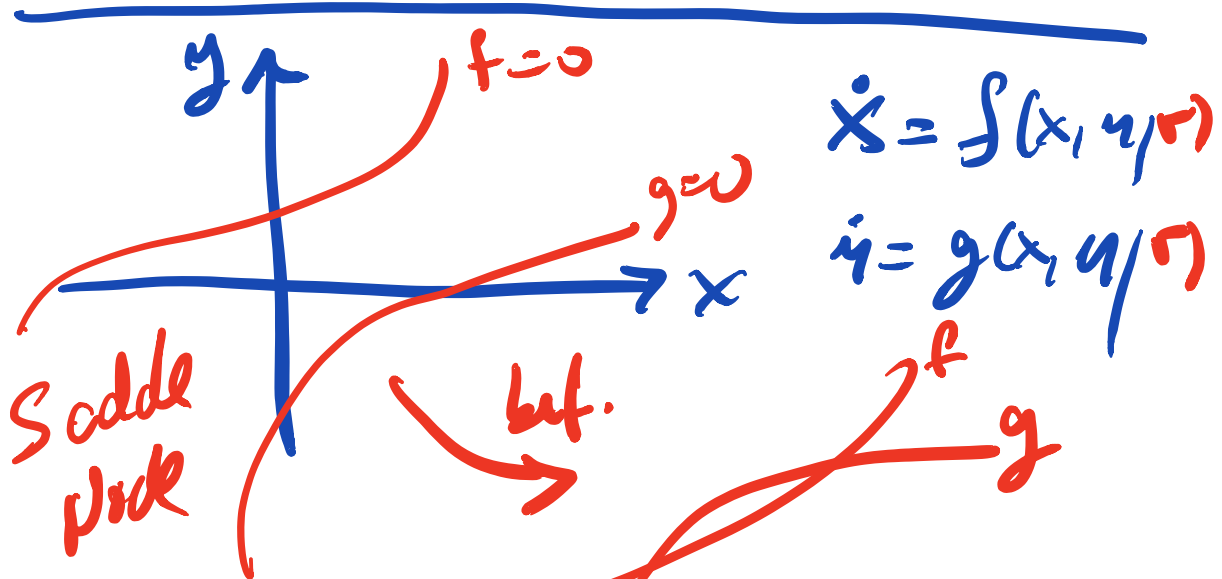
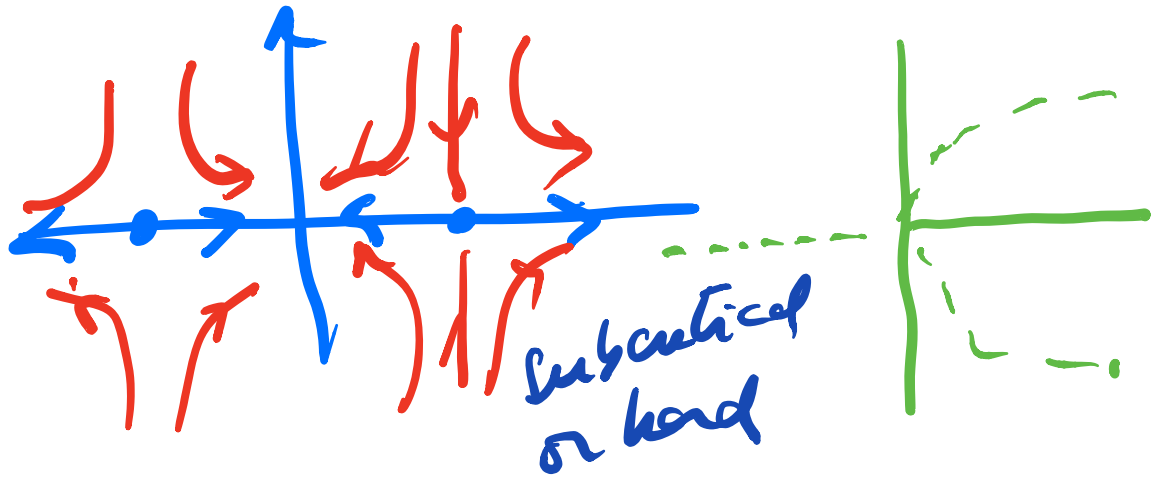


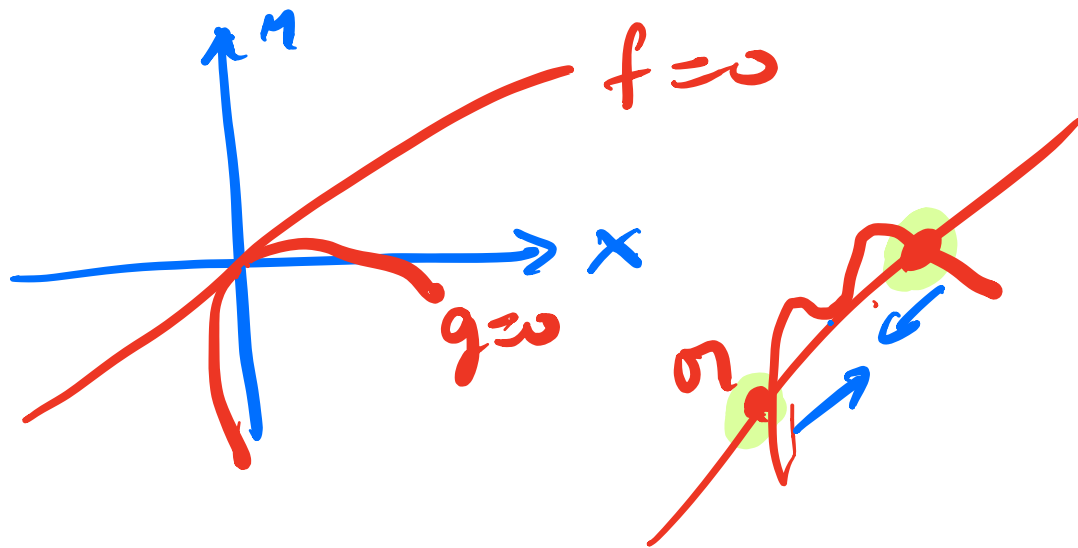
Supercritical
or subcritical



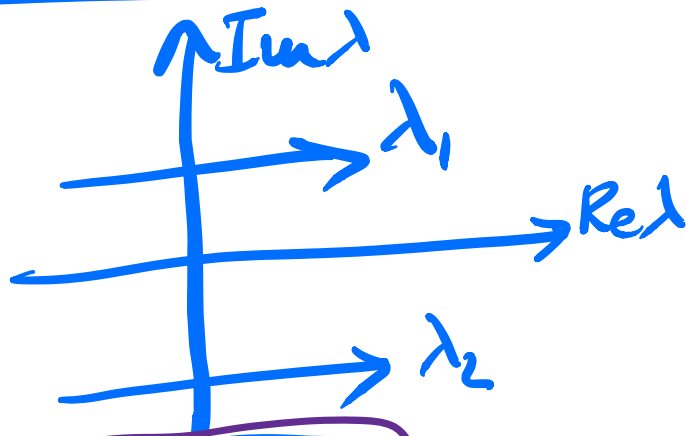
$$\dot{x} = rx - x^3$$

$r > 0$





Hopf bif.



C. Example

$$\begin{aligned} \dot{\Gamma} &= a\Gamma - \Gamma^3 \\ \dot{\Theta} &= 1 \end{aligned}$$

$$\lambda = \pm i + a$$

$a < 0$

origin stable

at 1 branch



stability

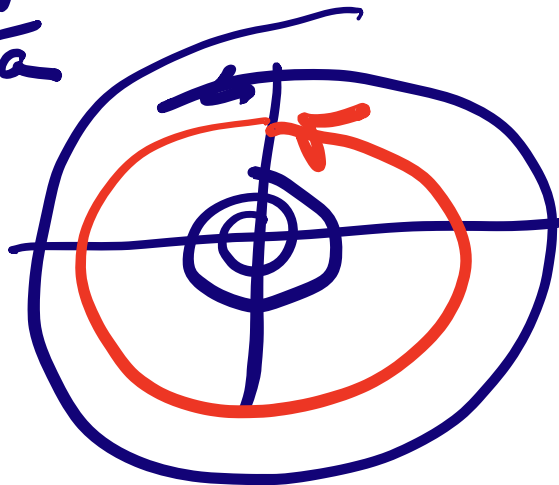


$a > 0$

$a > 0$



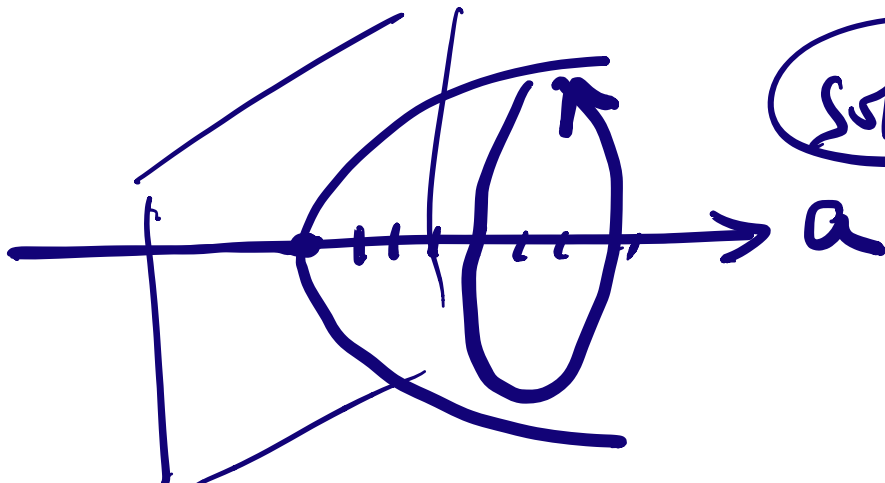
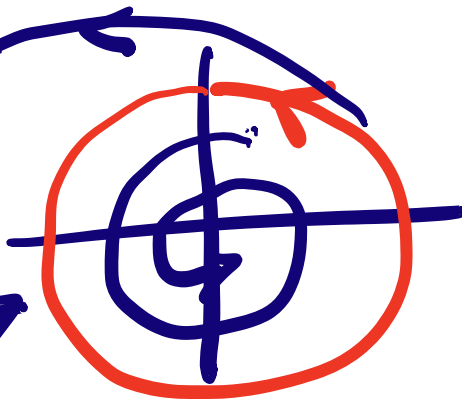
Soft-Hopf
or
bifurcation



$$\dot{r} = ar + r^3$$

$$\dot{\theta} = 1$$

$a < 0$

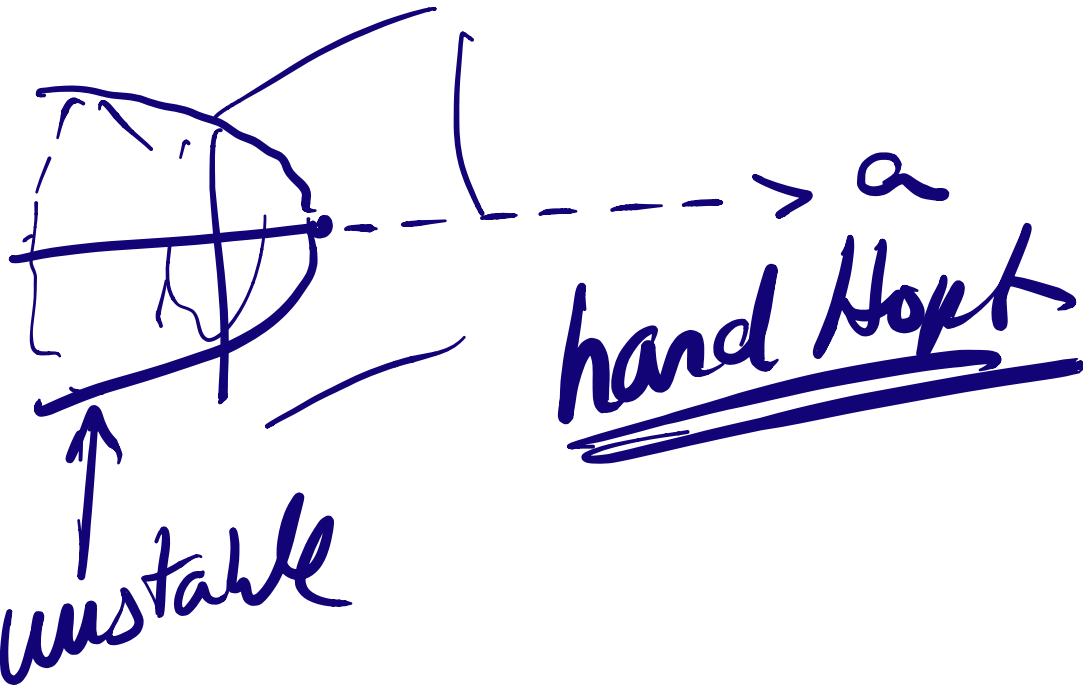


(soft)

limit cycle created

$$\text{Period} = O(1)$$

$$\text{Amplitude} \sim \sqrt{|F-E|}$$



$$\dot{r} = a r \pm r^2$$
$$\dot{\theta} = 1$$

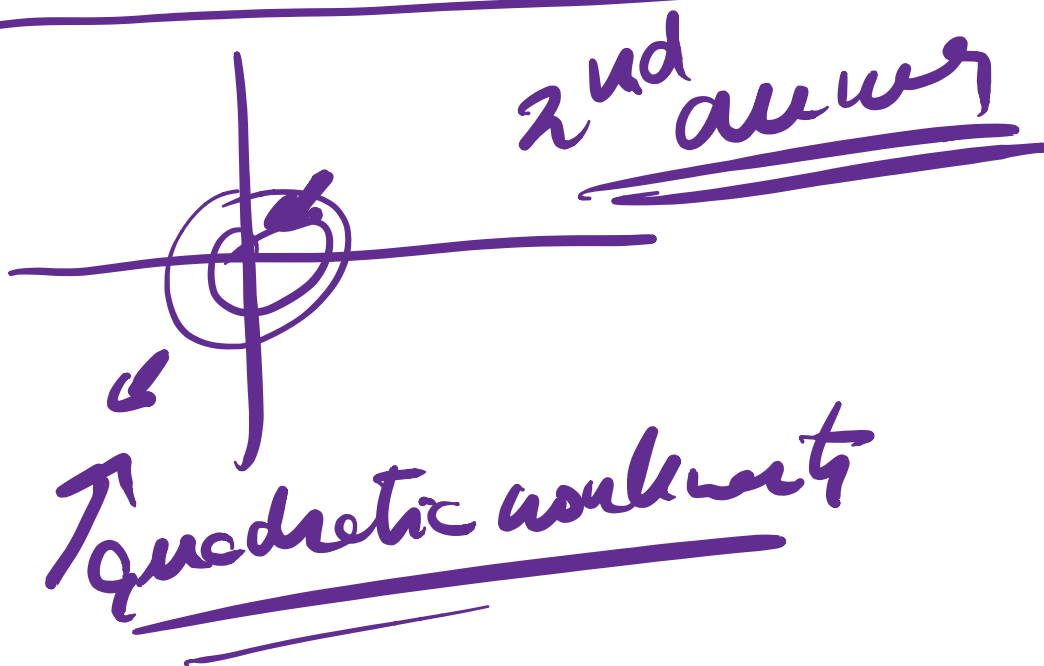
why not?

$$\dot{r} = f(r^2)$$

$$\dot{x} = f(r^2)x + y$$

$$\dot{y} = f(r^2)y - x$$

$$f(r) = \sqrt{x^2 + y^2}$$



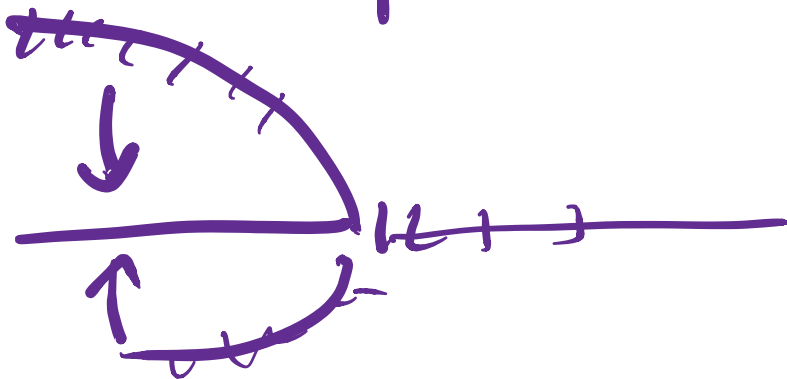
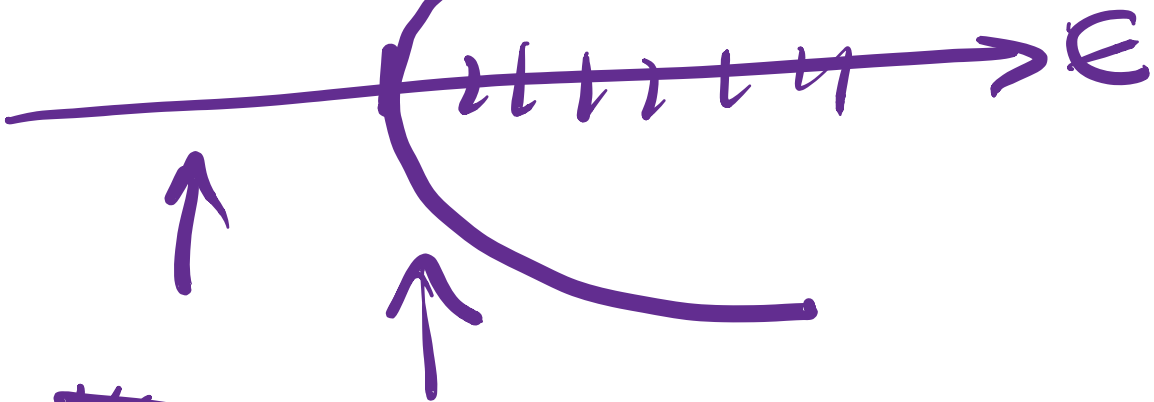
3rd answer

have linear in a switch
stability $\overrightarrow{1 \in x}$



size of growth part of
lever ten

~ \sqrt{E}



Canonical form of \dot{r}

$$\begin{aligned} \dot{r} &= a r \pm r^3 \\ \dot{\theta} &= 1 \end{aligned}$$

$$\dot{x} = f(x, y)$$

$$\dot{y} = g(x, y)$$

$$\ddot{x} + x + \epsilon x^2 = 0$$

$$x_0 + \epsilon x_1 + \frac{\epsilon^2}{2} x_2 + \dots$$

ϵ^2 \uparrow