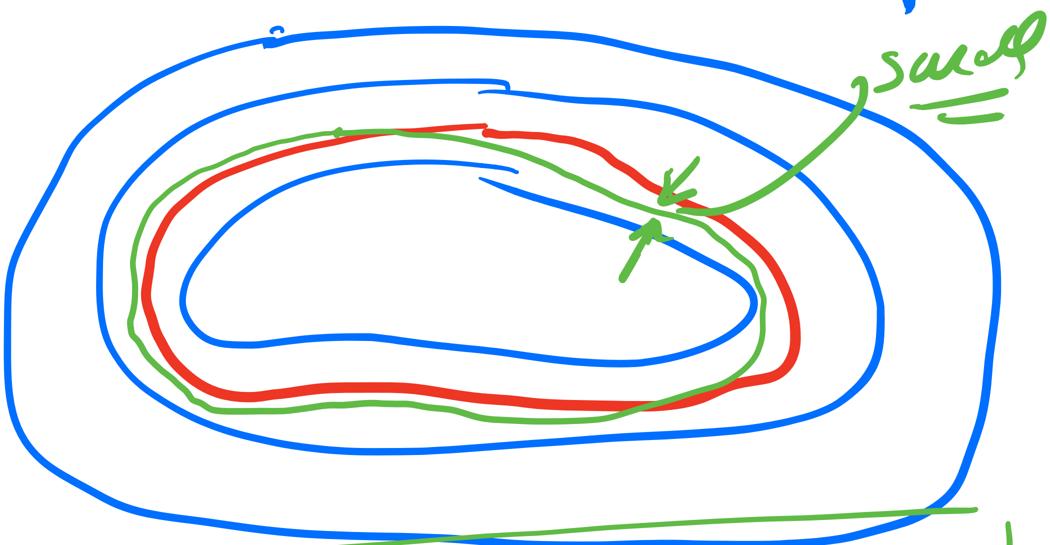
Poincaré - Linstead

Method to look for periodic solns.

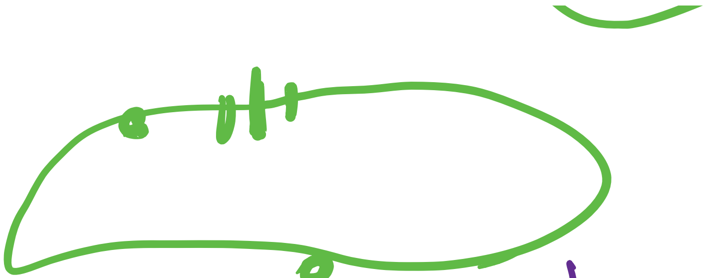


$$u = a\cos[\omega t] + \epsilon\phi(\omega t) + \dots$$

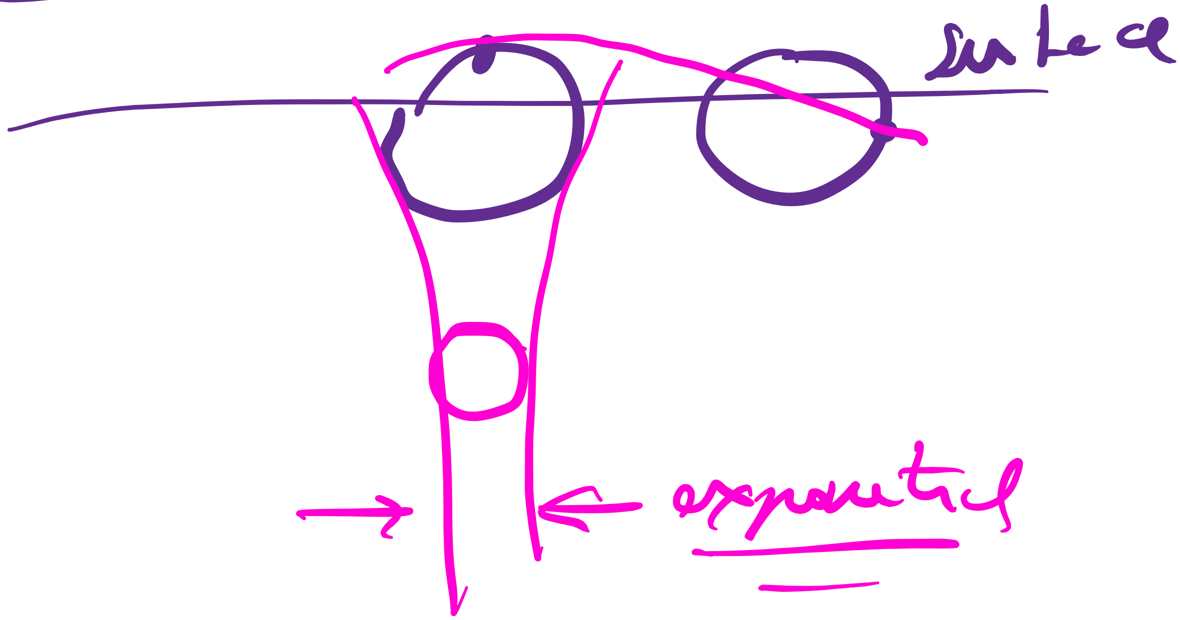
Find it.

$$\omega = \omega_0 + \epsilon\omega_1 + \dots$$

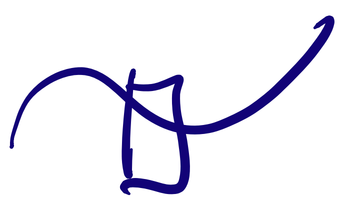
Stokes $\sim$ 1875



Stokes Drift

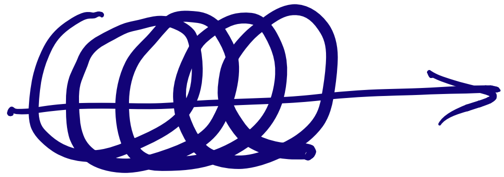


$a \cos(kx-\omega t)$



$\omega = \omega_0(k)$

$\omega = \omega_0(k) + \epsilon\, \omega_1(k) \ldots$



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Question what if zero order is nonlinear?

$$\ddot{x} + V'(x) = \epsilon f(x)\dot{x}$$

$$x = x_0(T) + \epsilon x_1(T) + \dots$$

$$T = \omega t, \quad \omega = \omega_0 + \epsilon \omega_1 + \dots$$

$$\omega^2 x'' - F(x) = \epsilon \omega f(x) x'$$

$$\omega_0^2 x_0'' - F(x_0) = 0$$

$$\boxed{x_0 = K(T, A)}$$

$$\omega_0 = \omega_0(A)$$

$$\omega_0^2 x_1'' - F'(x_0)x_1 = G$$

forcing

What does it mean to be resonant?

$$\mathcal{L} x_1 = G$$

$$\mathcal{L} = \frac{d^2}{d\tau^2} - F'(x_0)$$

$$\mathcal{L}\vec{v} = \vec{g}$$

Fredholm Alternative

This has a sln. iff.

$\vec{g}$ is orthogonal to all the solutions to the adjoint problem

$$\mathcal{L}^* w = 0 \Rightarrow \langle w, g \rangle = 0$$

$$\mathcal{L} = \boxed{\partial_T^2 + 1} \qquad \mathcal{L} w = 0$$

i.e. $w = a \sin T + b \cos T$

Need to know slns of

$$\left[\partial_T^2 - F'(x_0)\right] y = 0$$

$$Ax = y \qquad A^T z = 0$$

$$z \cdot y = 0$$

$$A^T z = 0 \qquad z^T A x = z^T y$$

$$(A^T z)^T x = 0$$

$$\omega^2 x_0'' + V'(x_0) = 0$$

$$\frac{1}{2}\omega^2 (x_0')^2 + V(x_0) = E$$

$$x_0 = x_0(E, T)$$

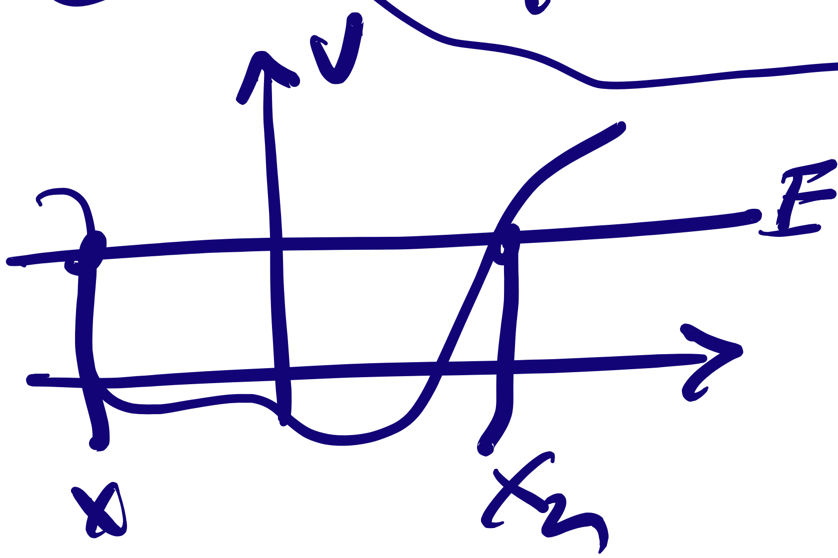
$$\omega^2(x_0) = 2(E - V(x_0))$$

$$x_0' = \pm \frac{2}{\omega}\sqrt{E - V(x)}$$

$$\frac{dx}{dt}$$

$$dt = \frac{\omega\, dx}{\sqrt{2(E - V(x))}}$$

$$\boxed{T = 2\omega \int_{x_1}^{x_2} \frac{dx}{\sqrt{2(E - V(x))}}}$$



$$T = 2\pi$$

$$\boxed{\omega = \frac{\pi}{\displaystyle\int_{x_1}^{x_2} \frac{dx}{\sqrt{2(E - V(x))}}}}$$