What happens if we do not include the slow time?

$0 < \epsilon \ll 1$

Consider the example $\ddot{x} + \epsilon x^2 \dot{x} + x = 0$, (A)

and try

$$x = x_0(t) + \epsilon x_1(t) + \epsilon^2 x_2(t) + \dots \quad (B)$$

Substituting into the equation and solving order by order in ϵ leads to:

$O(\epsilon^0)$ equation $\quad \ddot{x}_0 + x_0 = 0$

$$\therefore \; x_0 = A e^{it} + c.c.$$

where A is a complex constant and c.c. means "complex conjugate" (we want real solutions)

$O(\epsilon)$ equation $\quad \ddot{x}_1 + x_1 = -x_0^2 \dot{x}_0$

$$= -i A^3 e^{3it} - i|A|^2 A e^{it} + c.c.$$

Thus $x_1 = \frac{1}{8} i A^3 e^{3it} - \frac{1}{2} |A|^2 A \, t \, e^{it} +$

$$+ B e^{it} + c.c.$$

where B is a complex constant.

Note the term $t e^{it}$ and its c.c. in x_1

This term is growing, and when $\epsilon t = O(1)$ ~~[illegible]~~ we can no longer argue that ϵx_1 is a "small" correction to x_0

At this point the whole approximation breaks down. Thus (B) works only as long as ϵt is small

This approximation complete fails to capture the effect of the nonlinearity, which dissipates energy:

$$\frac{d}{dt}\left\{\frac{1}{2}\dot{x}^2 + \frac{1}{2}x^2\right\} = -\epsilon x^2 \dot{x}^2 < 0$$

The reason: it does not even try to do so!