Reduction to 1-D systems.

Earlier in the course we justified the introduction of 1-D systems (e.g. "bead on a wire" examples) as follows

1) Start with the equations $m\ddot{x} + \nu\dot{x} + V'(x) = 0$, where $F = -V'$ is the force.

2) Let L be a typical length scale for x (size of the devise) and F_0 be a typical force. Introduce the time scale $T = \nu L / F_0$, and use these scales to write the a-dimensional eqn

$$\epsilon\ddot{x} + \dot{x} + V'(x) = 0 \quad (A)$$

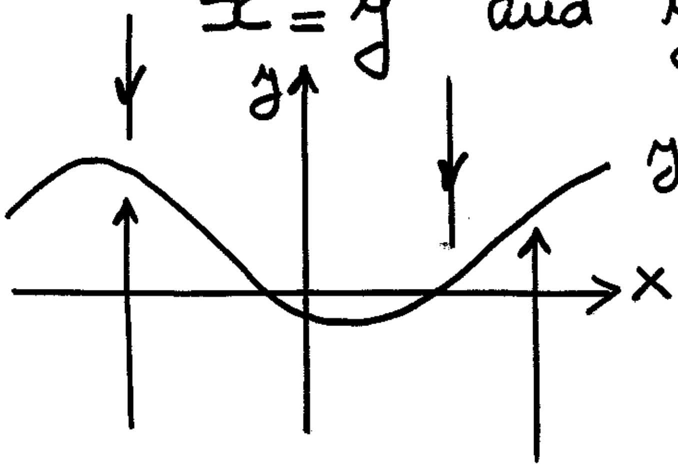
where $\epsilon = mF_0/\nu^2 L$.

3) Argue now that, for $0 < \epsilon << 1$ (large dissipation) we can neglect the $\epsilon\ddot{x}$ term. $\Longrightarrow$ 1-D eqn.

But we did not justify (3). In fact, generally you cannot drop the highest derivative just because it is multiplied by a small number. Example: from $\epsilon \ddot{x} + x = 1$ you cannot conclude $x \approx 1$ (i.e.: $x = A \sin\left[\frac{t - t_0}{\sqrt{\epsilon}}\right] + 1$ is the solution)!

Now we justify 3. Write (A) as a system

$$\dot{x} = y \quad \text{and} \quad \dot{y} = -\mu\left(y + V'(x)\right) \quad (B)$$



where $\mu = \frac{1}{\epsilon} >> 1$.

Then the same argument we used for the relaxation oscillations in van der Pol shows that the solutions rapidly approach the curve $0 = y + V(x)$. But $y = \dot{x}$! This is the HD system.

That is: dx/dt + V'(x) = 0.