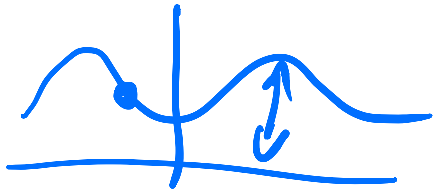
Reduction to 1-D systems



$\cancel{m\ddot{x}} + \nu\dot{x} + V'(x) = 0$

$\cancel{\epsilon\ddot{x}} + \dot{x} + V'(x) = 0$

ϵ "small"

Justify same technique as analyze limit cycle Relax. oscillations

Read notes

Next Topic

$$\ddot{x} - \epsilon\nu(1-x^2)\dot{x} + x = 0$$

vanderPol

$\boxed{\mu = \epsilon\nu}$

$0 < \epsilon << 1$

$\nu = \pm 1$

Case μ small

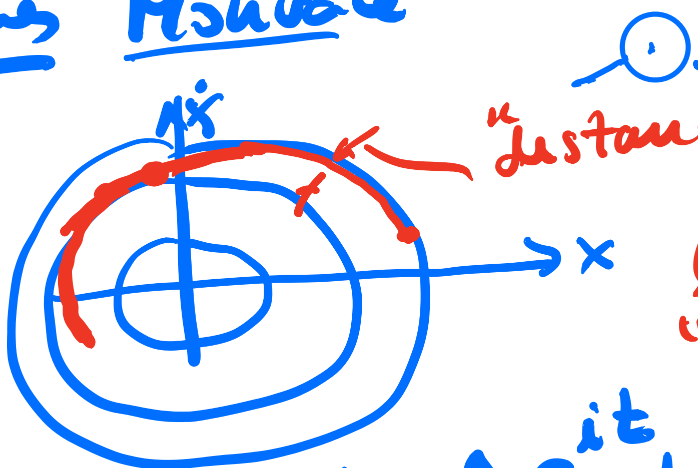
Asymptotic/perturbation methods

1. Two times / Multiple scales × not in book
2. Poincaré-Lindstead × book

Averaging × book

Duffing eqn $\boxed{\ddot{x} + x + \epsilon x^3 = 0}$

Two-timing Motivate



$$x \sim a \cos(t - t_0) = A e^{it} + c.c.$$

unperturbed system $\underline{A = \text{const}}$

$$\boxed{x \sim A(T)\, e^{it} + c.c.}$$

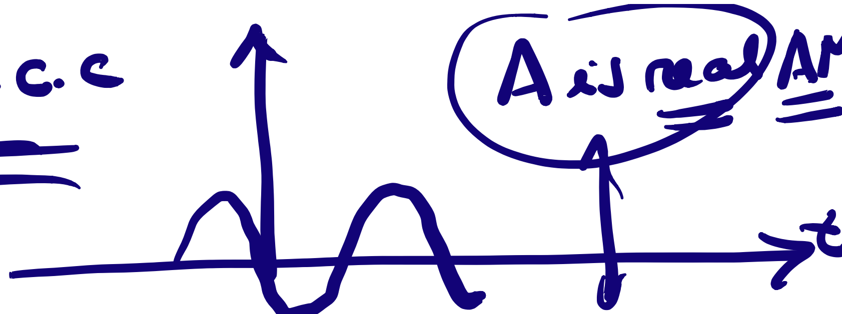
$\underline{T = \epsilon t}$ $\quad x = \boxed{x_0(t, T)}$

Main Idea Multiple scale

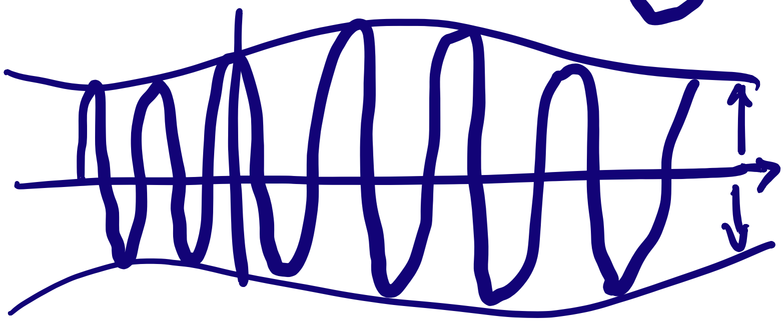
$$\boxed{x = x_0(t,T) + \epsilon x_1(t,T) + \epsilon^2 x_2(t,T)}$$

$$x = A(T)e^{it} + c.c$$

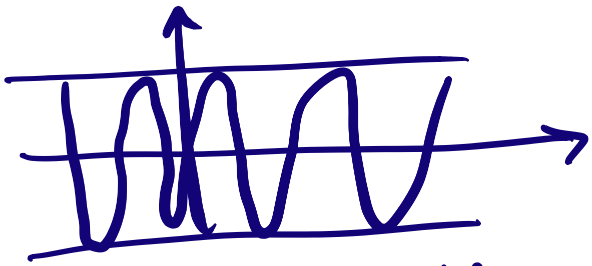
A is real AM



$|A| = \text{const.}$



AM



FM

General idea of the mechanism of 2 times

$$\boxed{\ddot{x} + x = \epsilon f(x, \dot{x})}$$

$$x = x_0(T,t) + \epsilon x_1(T,t) + \epsilon^2 x_2(T,t) + \dots$$

$T = \text{slow} = ?$

$' = \frac{\partial}{\partial t}$

$\bullet = \frac{d}{dt}$

$$x_0'' + x_0 = 0$$

$$x_0 = A(T)e^{it} + c.c.$$

$$x_1'' + x_1 = f(x_0, x_0') \qquad f_n = f_{-n}$$

$$= \sum f_n e^{int}$$

$$\boxed{\dot{X} = x' + \epsilon x_T}$$

$$\boxed{\ddot{X} = x'' + 2\epsilon x_T' + \epsilon^2 x_{TT}}$$

$n = \pm 1$ resonates!

$\Rightarrow$ term in $x_1 \propto t e^{it}$

If $f_1 \neq 0$ need $T = \epsilon t$

$$x_1'' + x_1 = -2 x_{0T}' + f(x_0, x_0')$$

What if $\boxed{f_1 = 0}$?

$$\ddot{X}_2 + X_2 = \text{Mess}$$

If "Mess" has e^{it} component

$\therefore$ take $T = \epsilon^2 t$

More complicated unperturbed

problem $\boxed{0 = Lx} + \epsilon N$

$Lx = 0$
has periodic
solns

$$\boxed{Lx_0 = 0}$$

$$\boxed{Lx_1 = N_1}$$