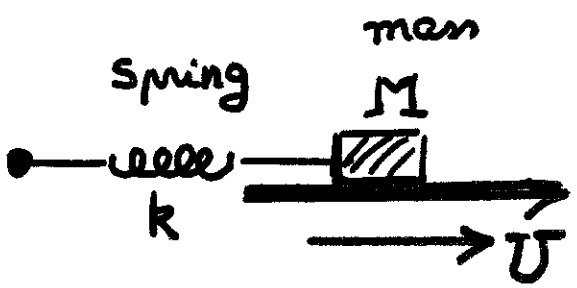
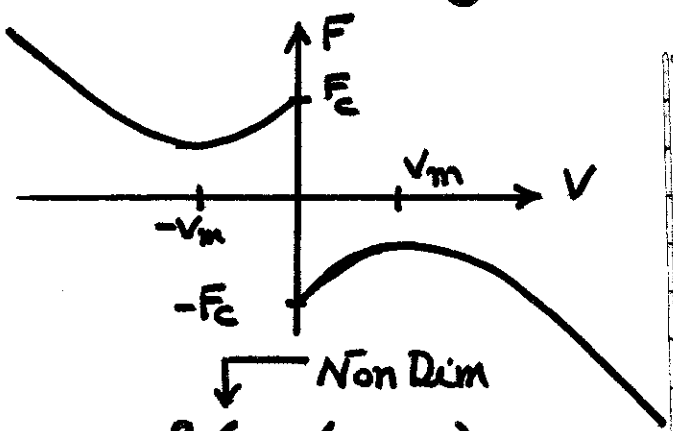
Relaxation oscillations due to friction

Refrigerator Pushing



conveyor belt

Toy Model



Friction function as function of velocity. The existence of a static critical load F_c and a critical velocity V_m are important. $x=0$: spring relaxed

Non Dim

$$F = F_c \, f(V/V_m)$$

Equation $\quad M\ddot{x} = -kx + F(\dot{x} - U)$

Equillibrium $\quad x = x^*$, where $kx^* = F(-U)$

so, write $\quad x = x^* + X$

Then $\quad M\ddot{X} = -kX + F(\dot{X} - U) - kx^*$

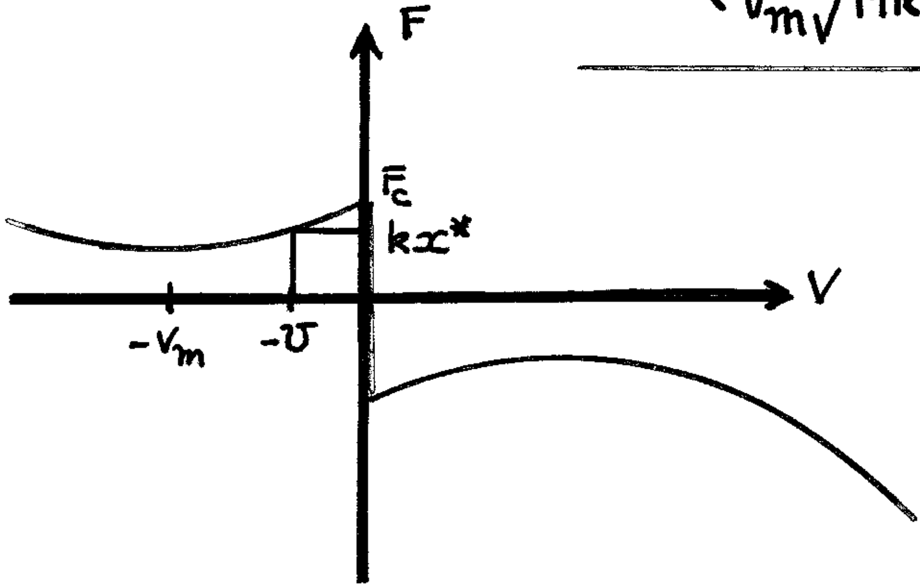
Assume now: $\quad 0 < U < V_m \;, \; U = O(V_m)$

and nondim. $\quad x = (F_c/k)\,\tilde{x}\,; \quad x^* = \frac{F_c}{k}\tilde{x}^*\,;$

$$t = \underbrace{\sqrt{\frac{M}{k}}}_{\text{mass-spring response time.}}\,\tilde{t}\,; \quad U = V_m \tilde{U}$$

Then: $$\ddot{x} = -x + f\left(\frac{F_c}{V_m\sqrt{Mk}}\,\dot{x} - U\right) - x^*$$

drop ~ in nondim variables.

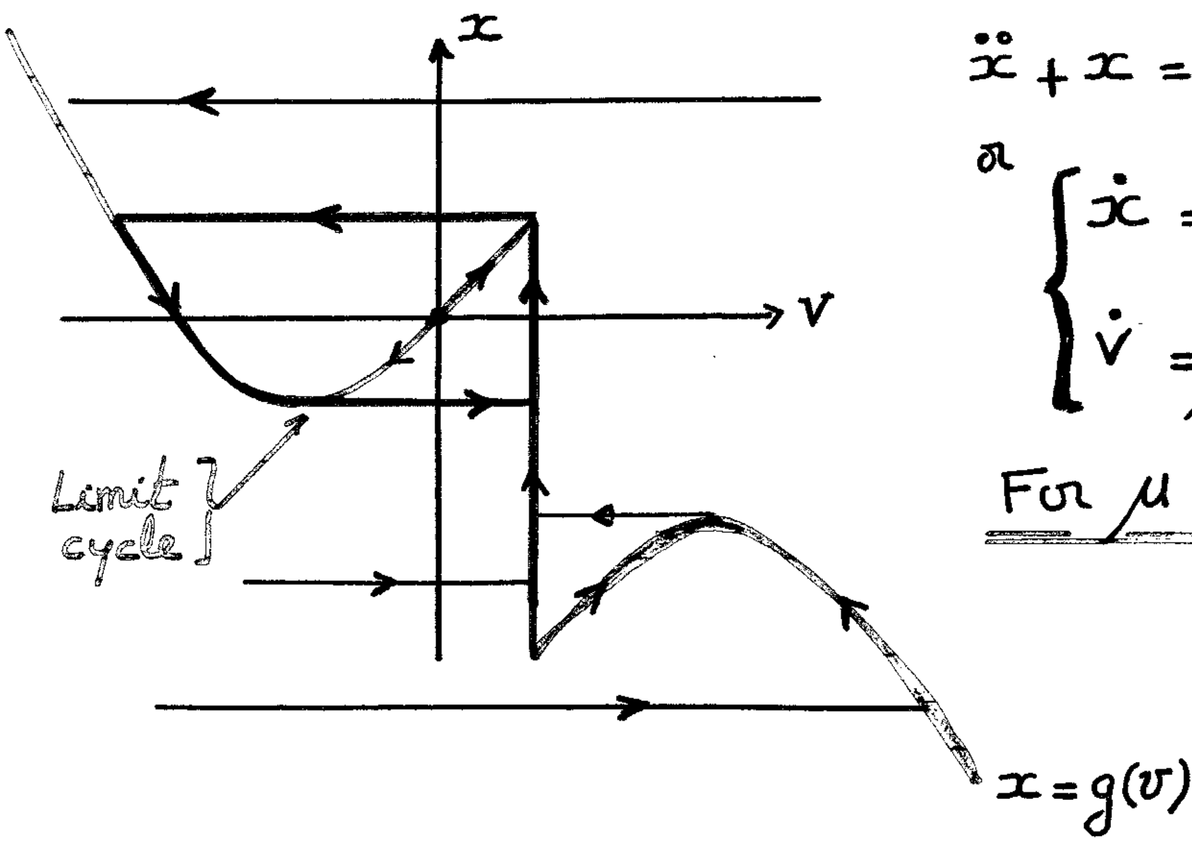


$$\boxed{\mu = \frac{F_c}{V_m\sqrt{Mk}}}$$

Non Dim parameter

In non-dim version: $F_c = 1$, $V_m = 1$, $F = f$
$k = 1$, $0 < U < 1$.

Let now $g(v) = f(v - U) - x^*$



$$\ddot{x} + x = g(\mu\dot{x})$$

or
$$\begin{cases} \dot{x} = \frac{1}{\mu} V \\ \dot{V} = \mu(g(V) - x) \end{cases}$$

For $\mu >> 1$

Thus, the conditions under which a relaxation limit cycle occur are:

$$0 < U < v_m$$

Mean pushing velocity in critical range

and $$F_c \gg v_m \sqrt{Mk}$$

This seems to indicate that the mass cannot be very large. How-ever $F_c \approx M a$

a typical acceleration

So get $\sqrt{M} \gg v_m \sqrt{k} / a$

F_c depends on weight!