Trapping regions

$$\boxed{\ddot{x} + \mu(x^2-1)\dot{x} + x = 0}$$

Has a stable limit cycle for any $\mu > 0$

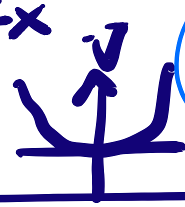
$$\ddot{x} + \frac{d}{dt}\left[\mu\left(\frac{x^3}{3} - x\right)\right] + x = 0$$

Liénard systems

$$0 = \ddot{x} + \frac{d}{dt} f(x) + V'(x)$$

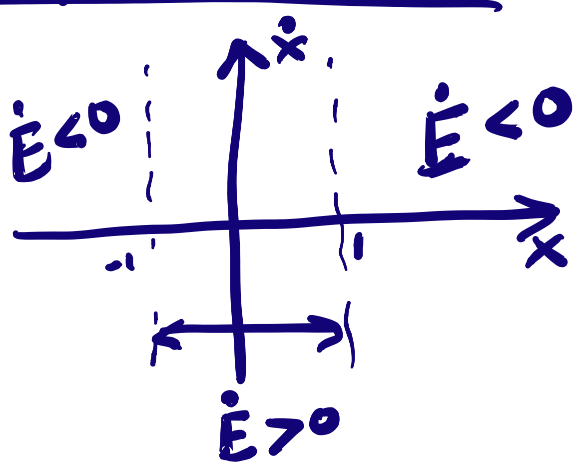
① f looks like $\frac{1}{3}x^3 - x$

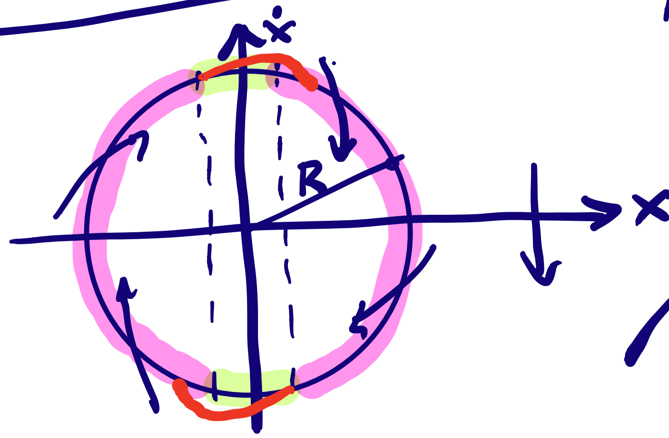
② V " " $\frac{1}{2}x^2$

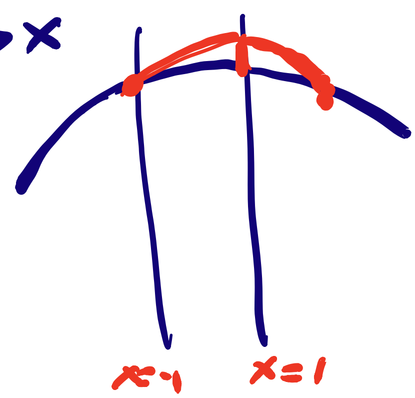


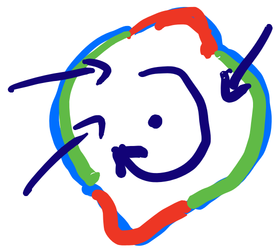
$$E = \frac{1}{2}\dot{x} + \frac{1}{2}x^2$$

$$\boxed{\frac{dE}{dt} = -\mu(x^2-1)\dot{x}^2}$$









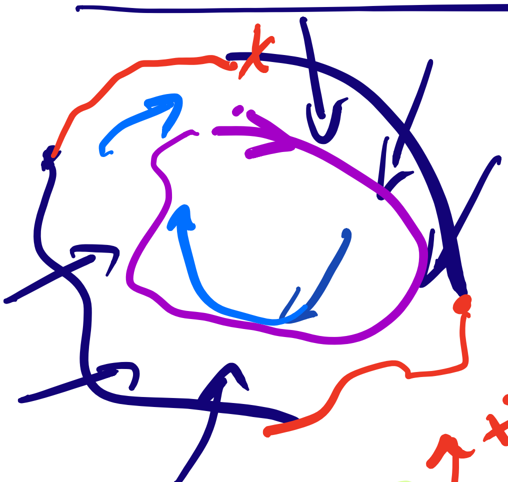
Linearized eqn

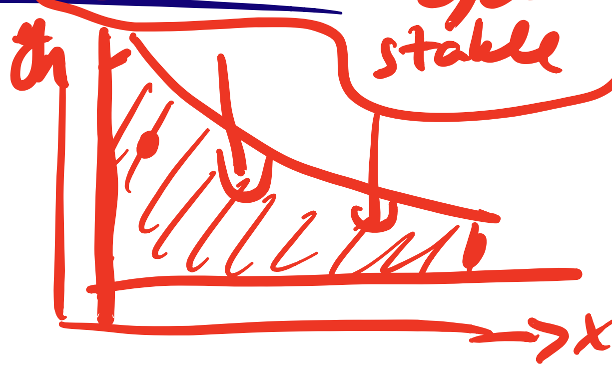
$$\ddot{x} - \mu\dot{x} + x = 0$$

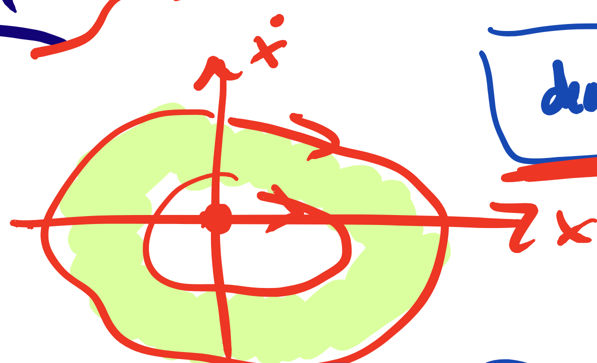
near $x = \dot{x} = 0$

Trapping + Simple unstable source $\Longrightarrow$ periodic orbit must exist

But: is it a limit cycle stable





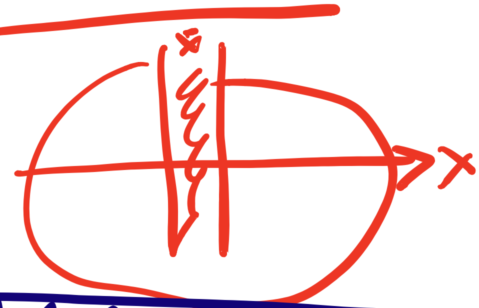


$$\boxed{\operatorname{div}(g\cdot\vec{v}) \neq 0}$$

$$0 < \iint_A \nabla\cdot(g\vec{v})\, dx\, d\dot{x} = \oint_{\partial A} g\vec{v}\cdot\hat{n}\, ds = 0$$

$$\boxed{E = \frac{1}{2}\dot{x}^2 + V(x)}$$

$$\dot{E} = -f'(x)\dot{x}^2$$



$$\ddot{x} + \mu(x^2-1)\dot{x} + x = 0$$

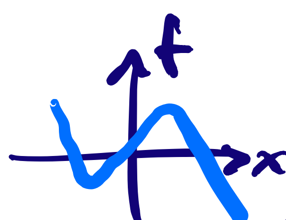
vdP

$\mu > 0$

$\mu >> 1$ $\mu << 1$ relaxation oscillations

$$\frac{d}{dt}\left[\dot{x} - \mu f(x)\right] + x = 0$$

$$\boxed{f = -\frac{1}{3}x^3 + x}$$



$$\boxed{x = \dot{y}}$$

$$\boxed{\dot{x} - \mu f(x) + y = 0}$$

Rayleigh Equ

$$\boxed{\ddot{y} - \mu f(\dot{y}) + y = 0}$$

$$\boxed{\begin{aligned}\dot y &= x \\ \dot x &= \mu f(x) - y\end{aligned}}$$

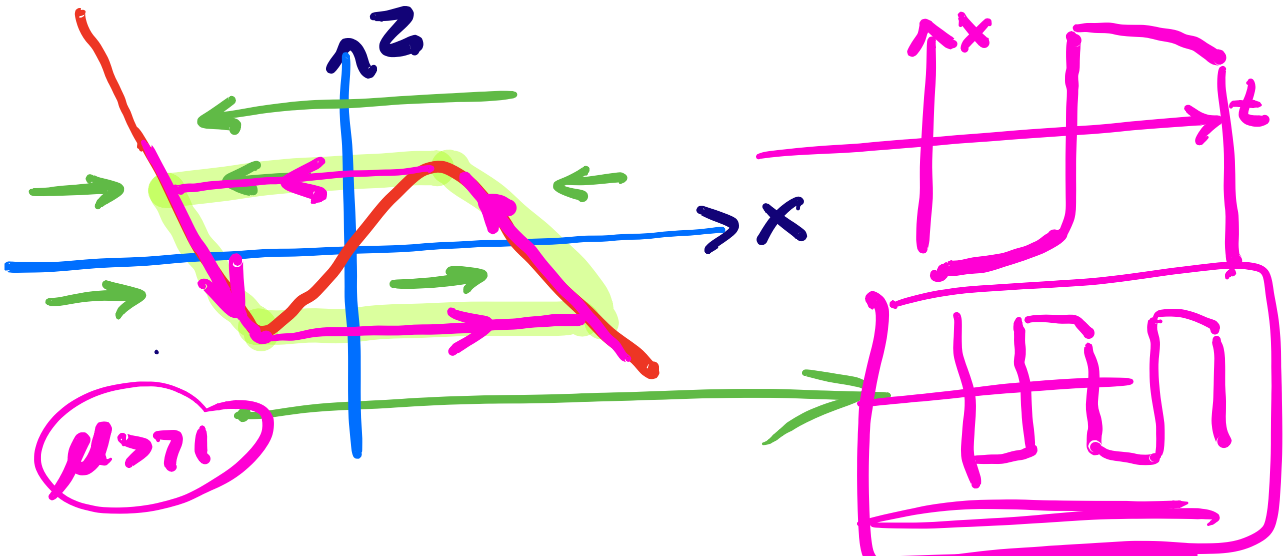
$$\boxed{y = \mu z}$$

$$\boxed{\begin{aligned}\dot z &= \frac{1}{\mu} x \\ \dot x &= \mu (f(x) - z)\end{aligned}}$$

$$\boxed{\begin{aligned}\dot u &= \frac{1}{\mu} g(u,v) \\ \dot v &= \mu f(u,v)\end{aligned}}$$

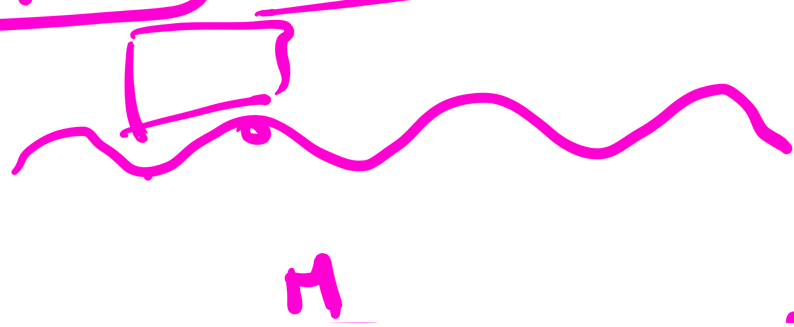
$\mu >> 1$

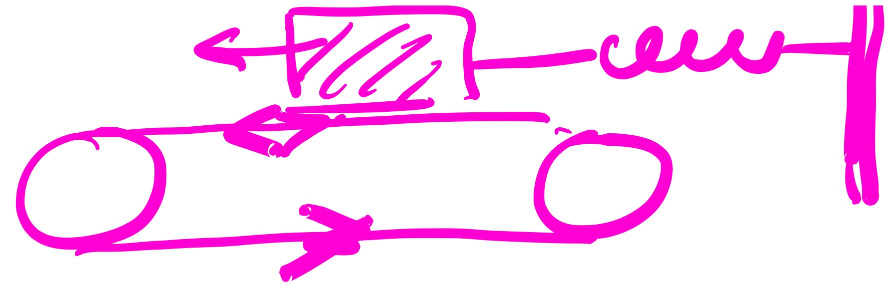
$z \sim f(x)$

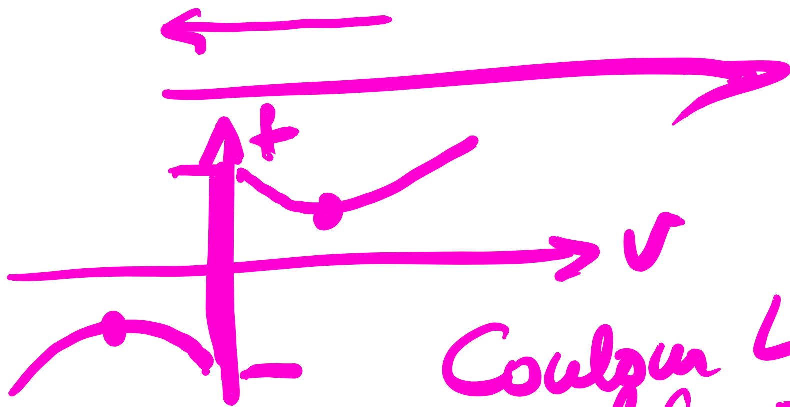


Mechanical example

Stick-Slip







Coulomb Law of friction