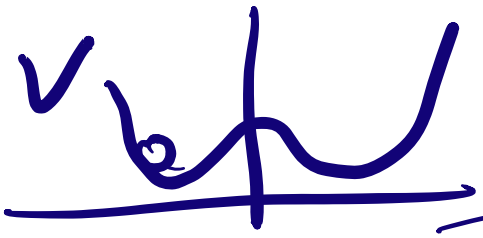
Example (Cont. Lyp. 1)



$$m\ddot{x} = -\frac{\partial V}{\partial x}$$

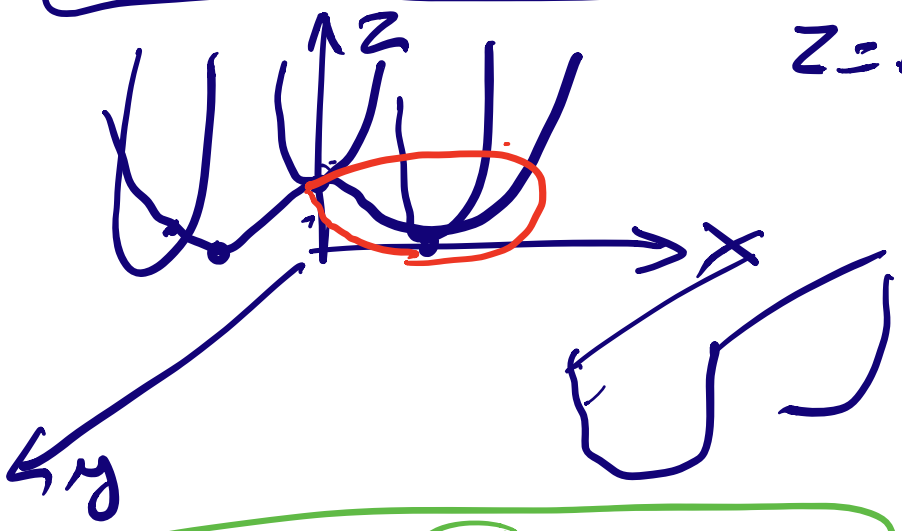
$$V = 1 - x^2 + x^4$$

$$E = \frac{1}{2}m\dot{x}^2 + V(x)$$

$$= \frac{1}{2m}y^2 + V(x)$$

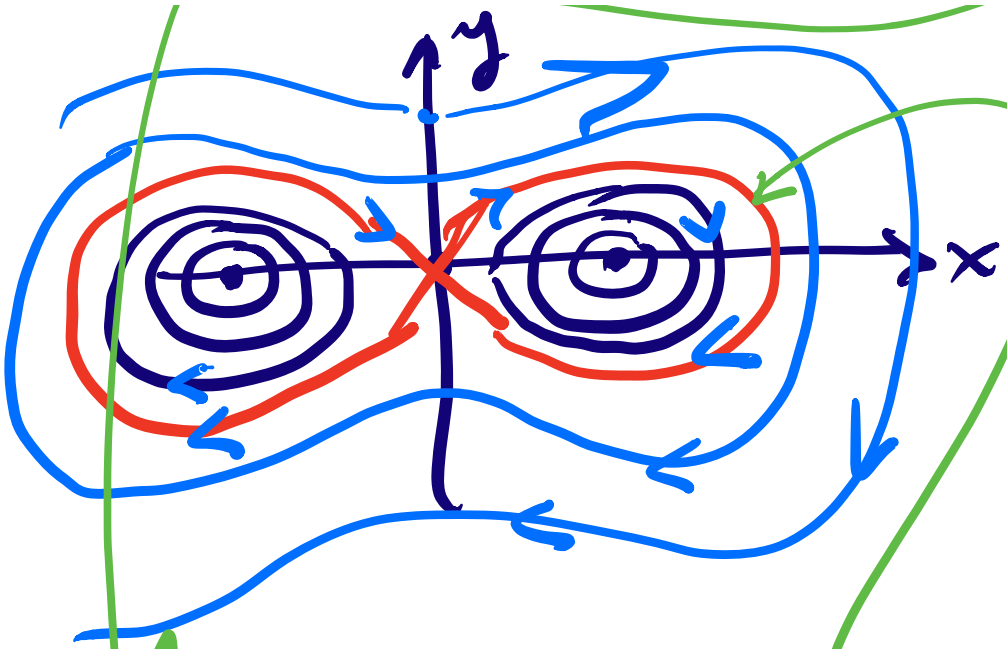
$$m\dot{x} = y$$

$$\dot{y} = -\frac{\partial V}{\partial x}$$



$$z = E(x, y)$$

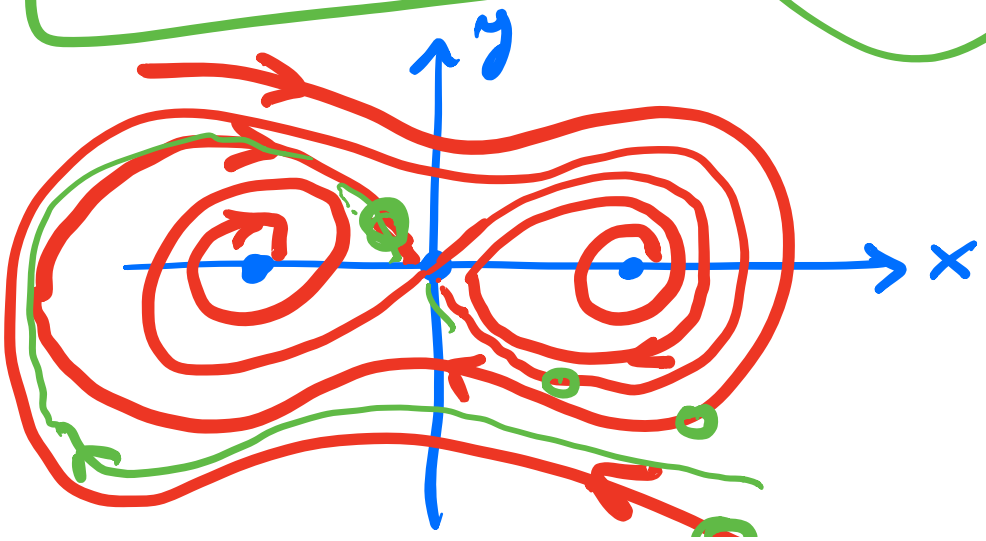
$$m\ddot{x} + k\dot{x} + \frac{\partial V}{\partial x} = 0$$



homo saddle connection

$$\frac{d}{dt}E = -\nu\dot{x}^2$$

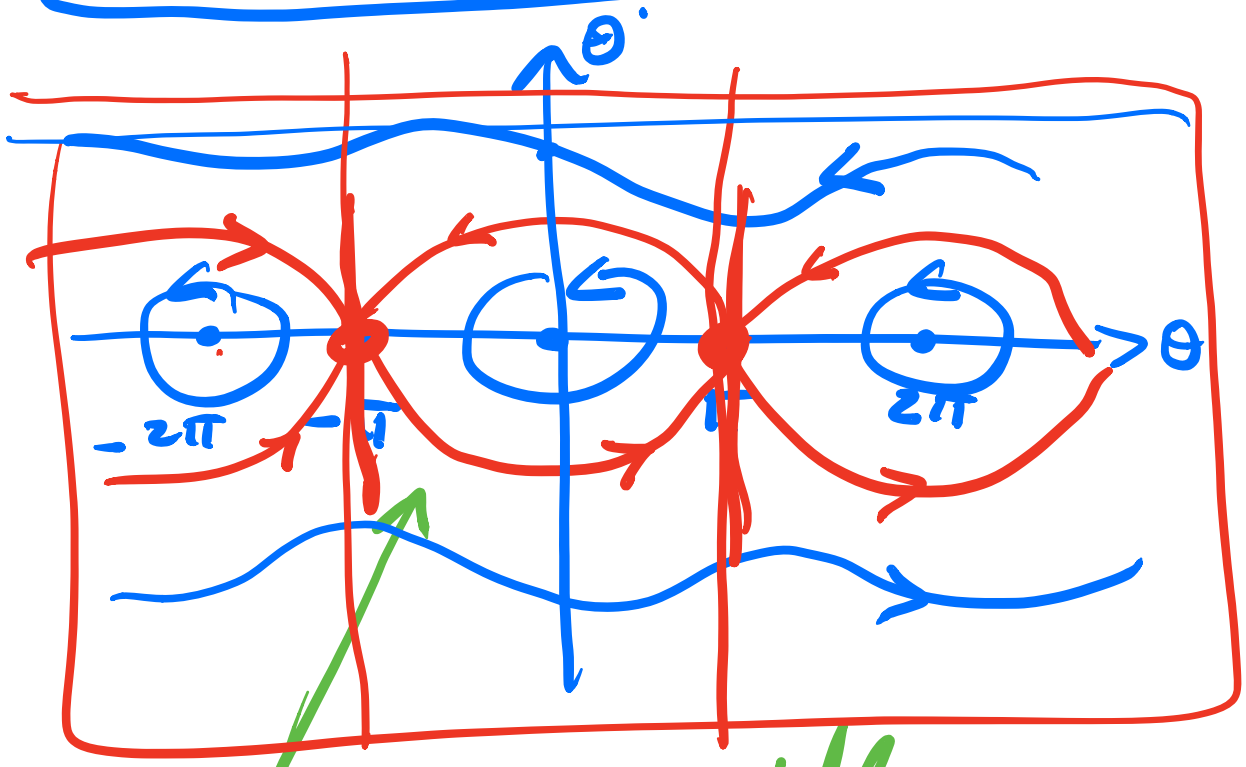
$$\frac{dE}{dt} \leq 0$$



$$\ddot{\theta} + \sin\theta = 0$$

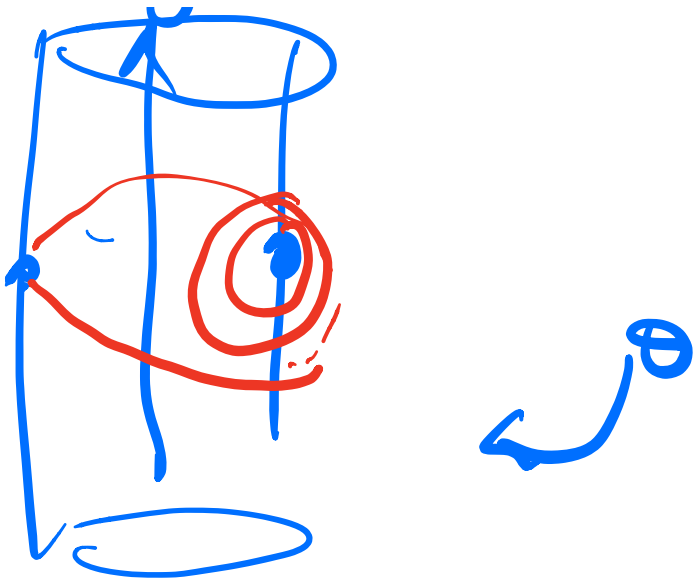
$$\sin\theta = \frac{\partial}{\partial\theta}(1-\cos\theta)$$

$$\boxed{E = \frac{1}{2}\dot{\theta}^2 + (1-\cos\theta)}$$



heteroclinic saddle con.

$$\ddot{\theta} + \nu\dot{\theta} + \sin\theta = 0$$



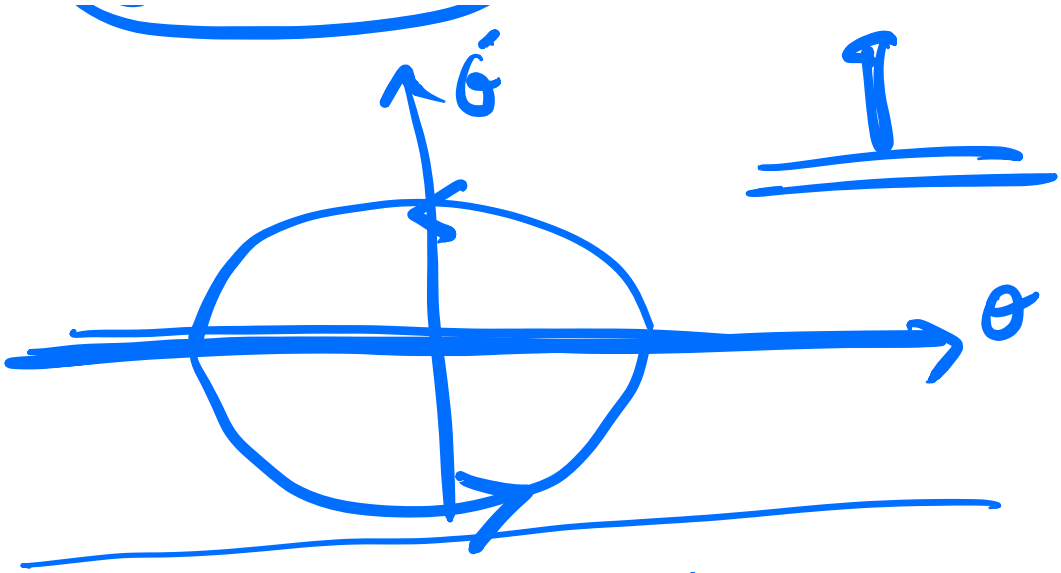
$$m\ddot{x} + \frac{\partial V}{\partial x} = 0 \qquad V = V(x)$$

$$m\dot{x} = y$$

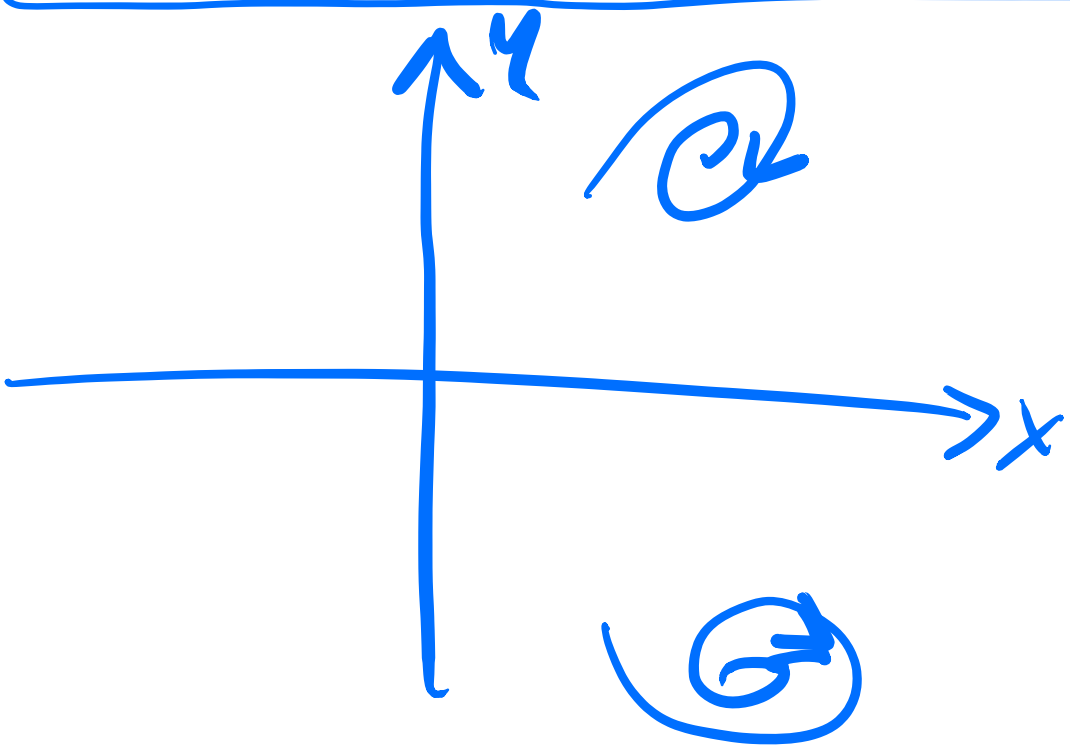
$$\dot{y} = -\frac{\partial V}{\partial x}$$

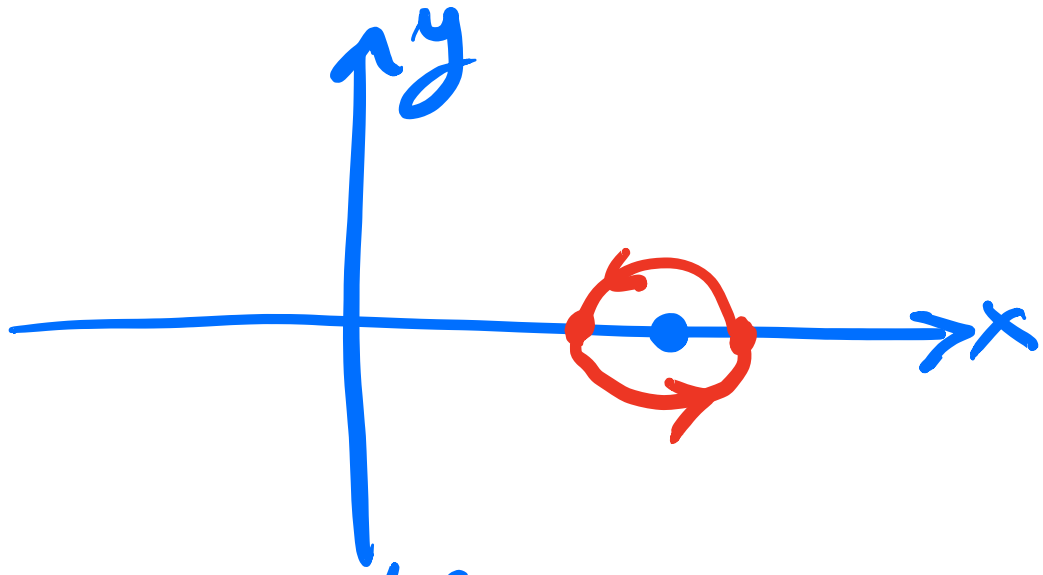
$t \to -t \qquad \tilde{x} = x(-t)$

$(y \to -y)$ time reversible

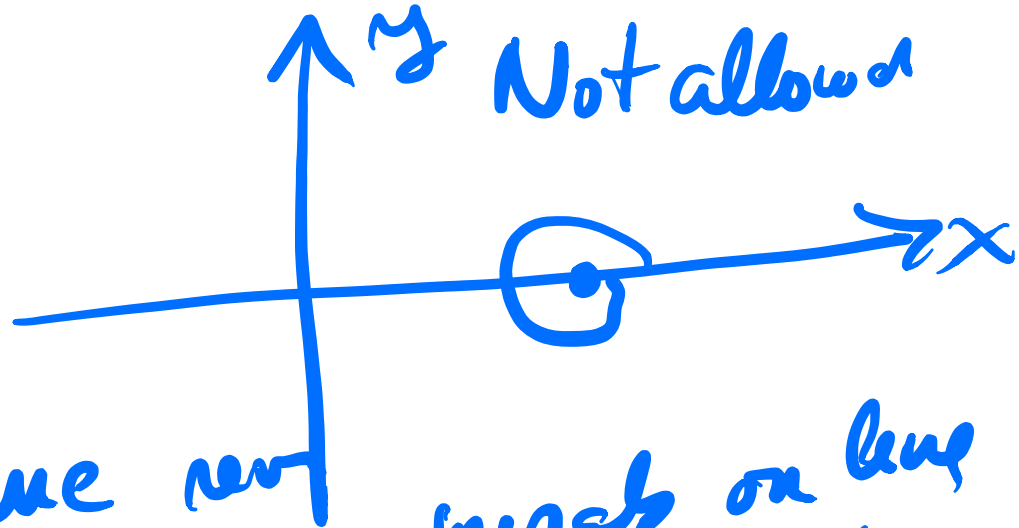


Conservative $\neq$ Irreversible





Time reversible +
linear center on axis of
symmetry (i.e. $y=0$)

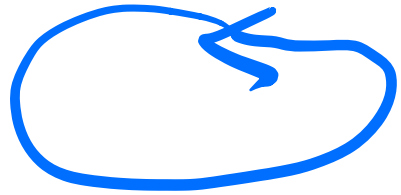


Not allowed

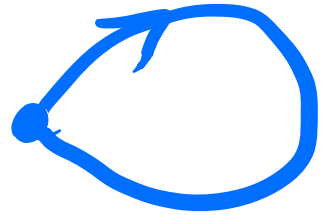
Time rev
→ no spirals on line
of symmetry

No nodes either

Two types of "closed" orbits

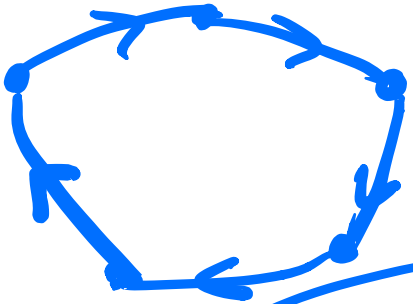


periodic near orbits



homoclinic connection (Two orbits)

Three orbits

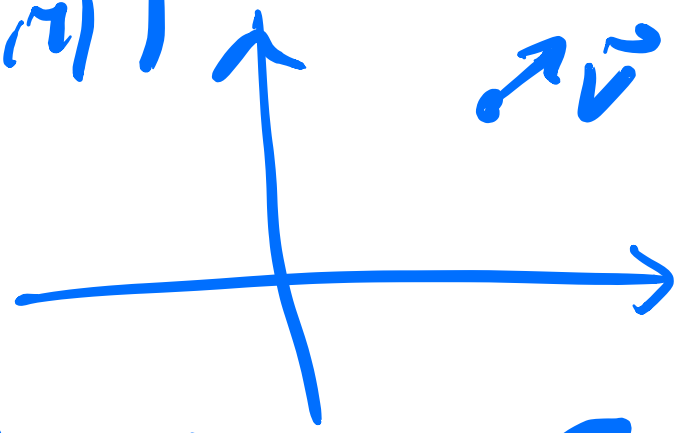


cycle graphs

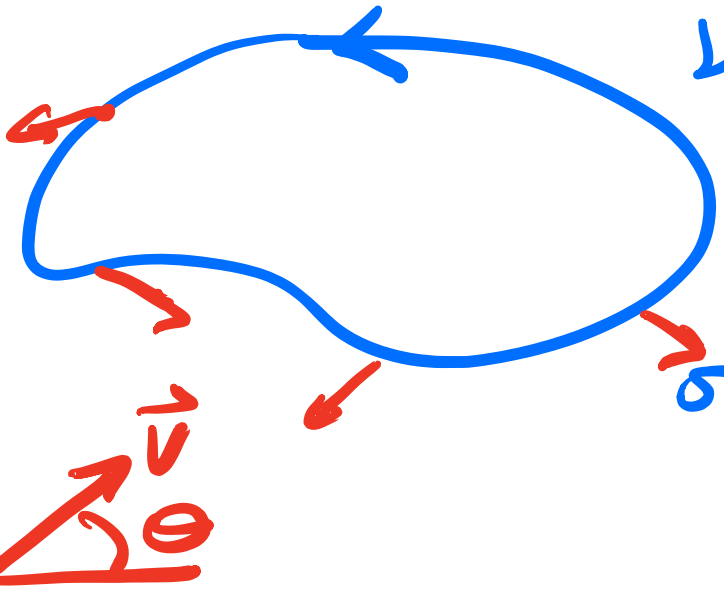
Index Theory

$\dot{x} = f(x,y)$
$\dot{y} = g(x,y)$

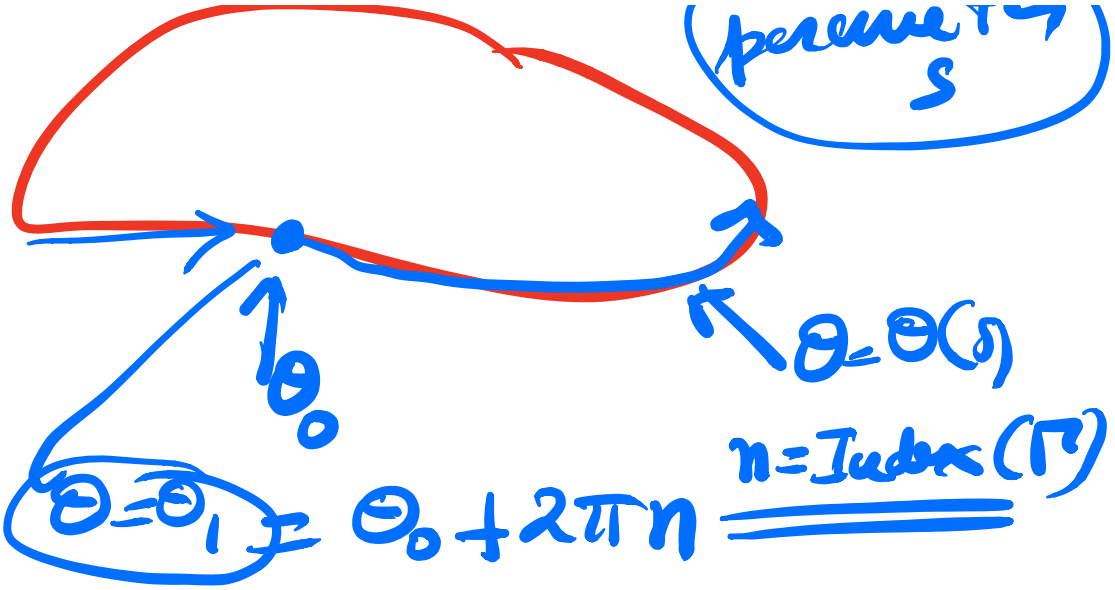
$\vec{V} = (f, g)$



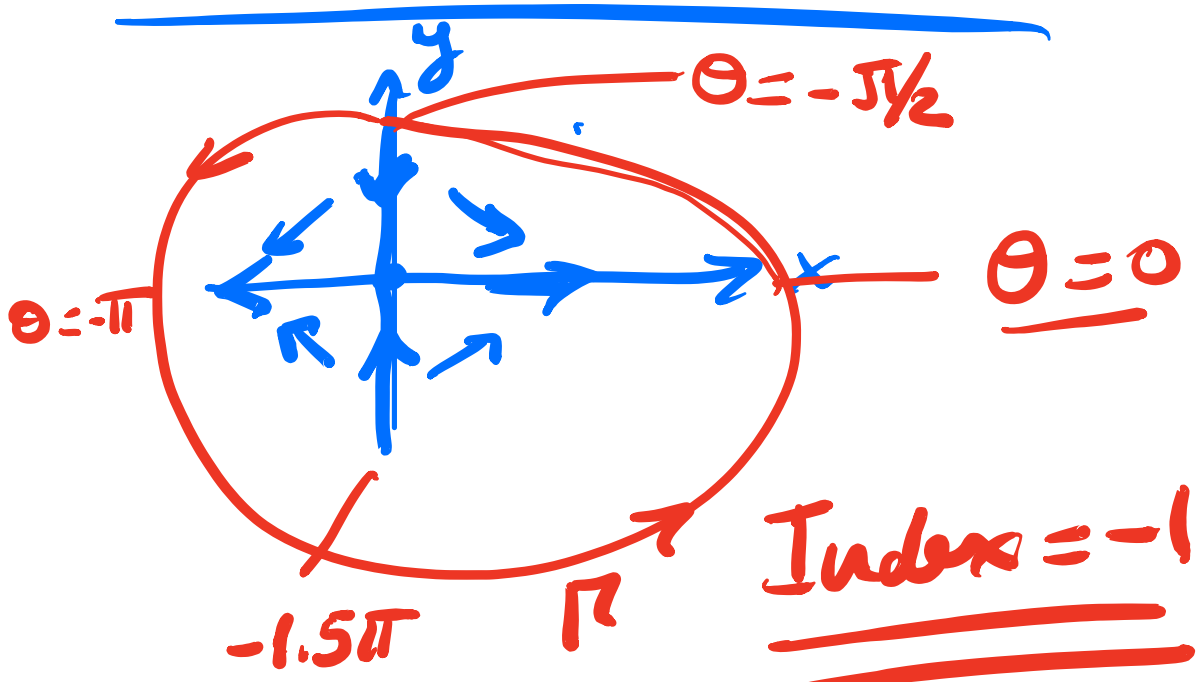
Take closed curve Γ



such that no crit. points on Γ



Say Γ encloses a saddle

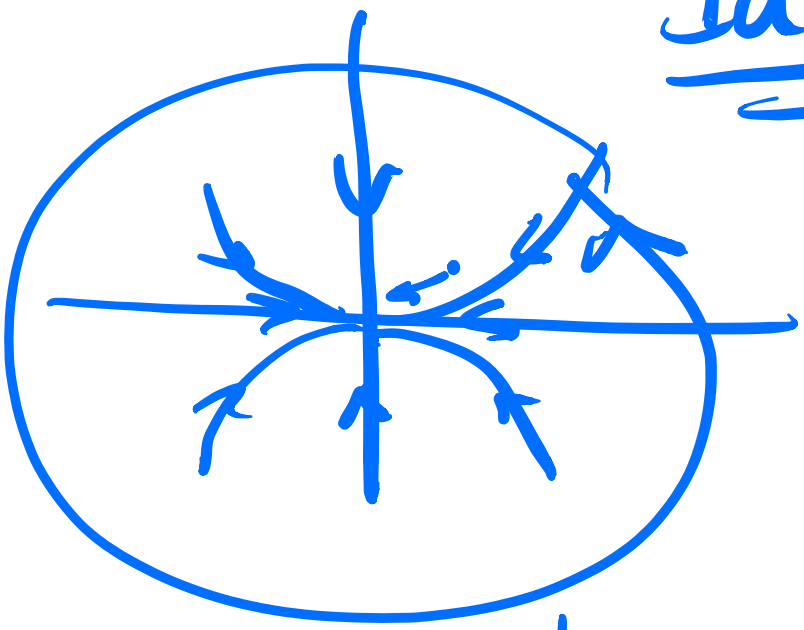


Index $= -1$

Saddle

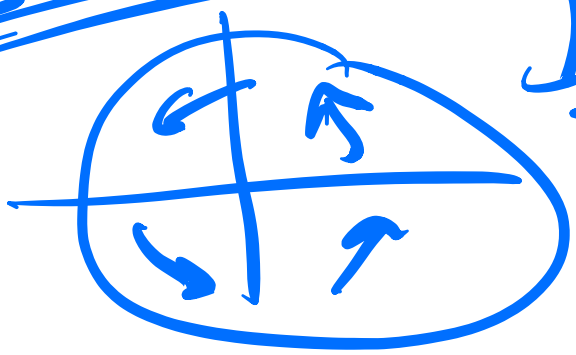
Index Node

Index = 1



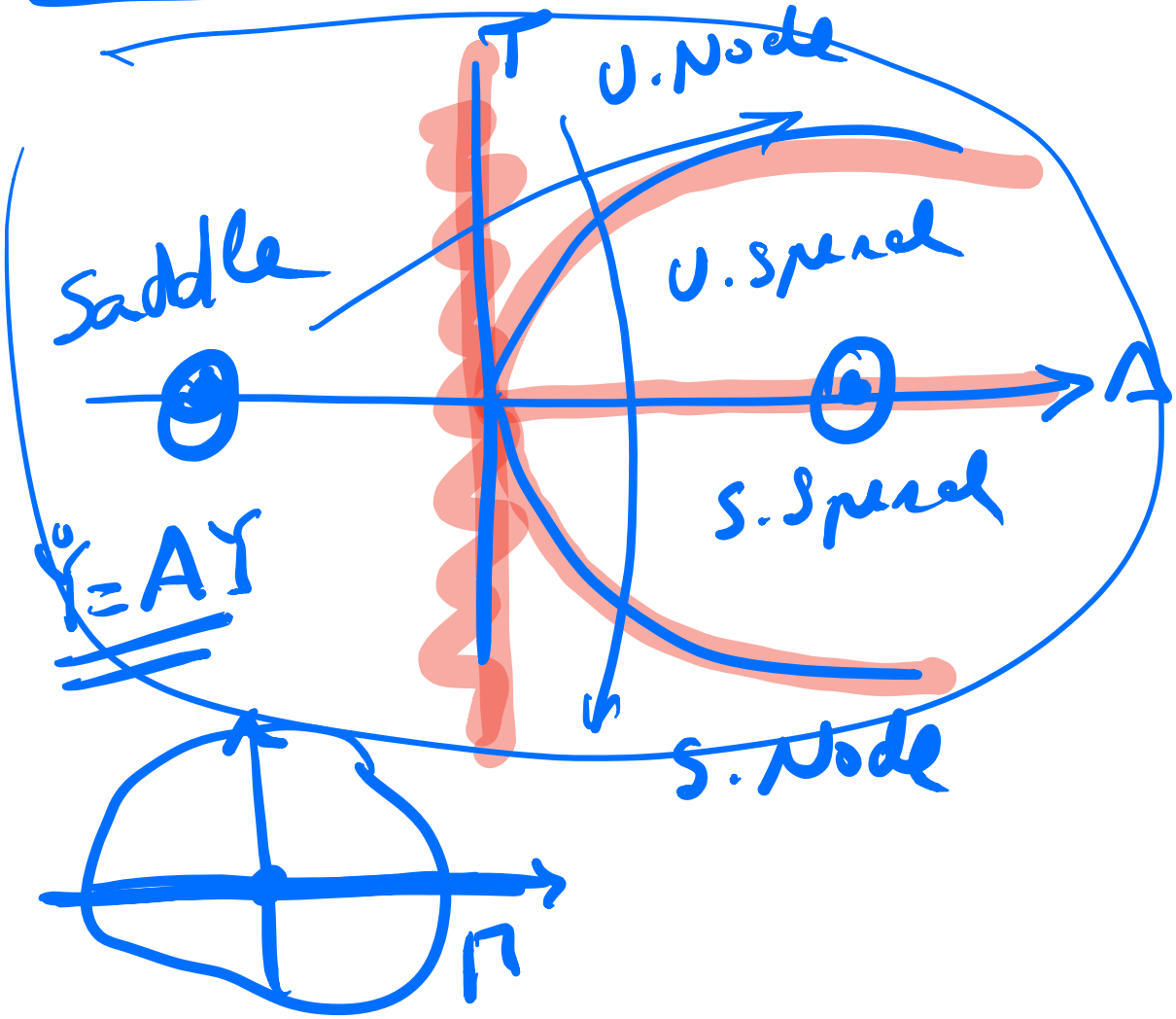
Index Center

Index = 1



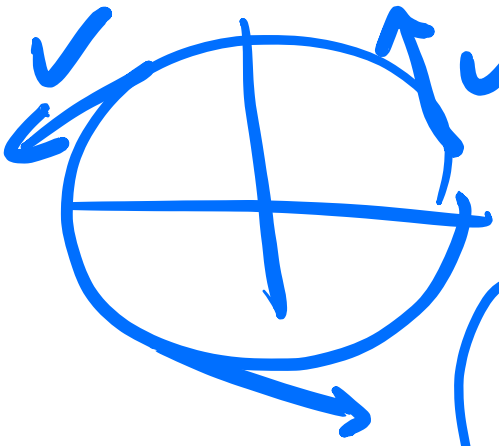
Index Sum = 1

Index does not change if Γ is changed so that Γ never has a c.p. on it.

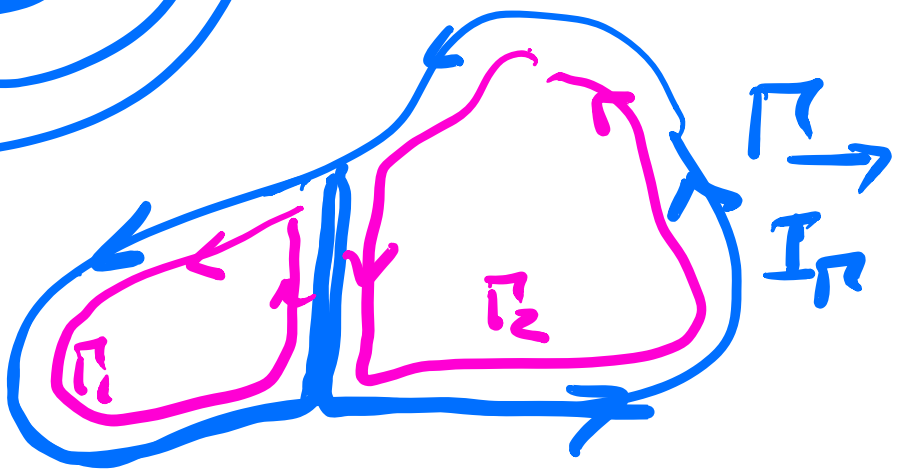


Next index

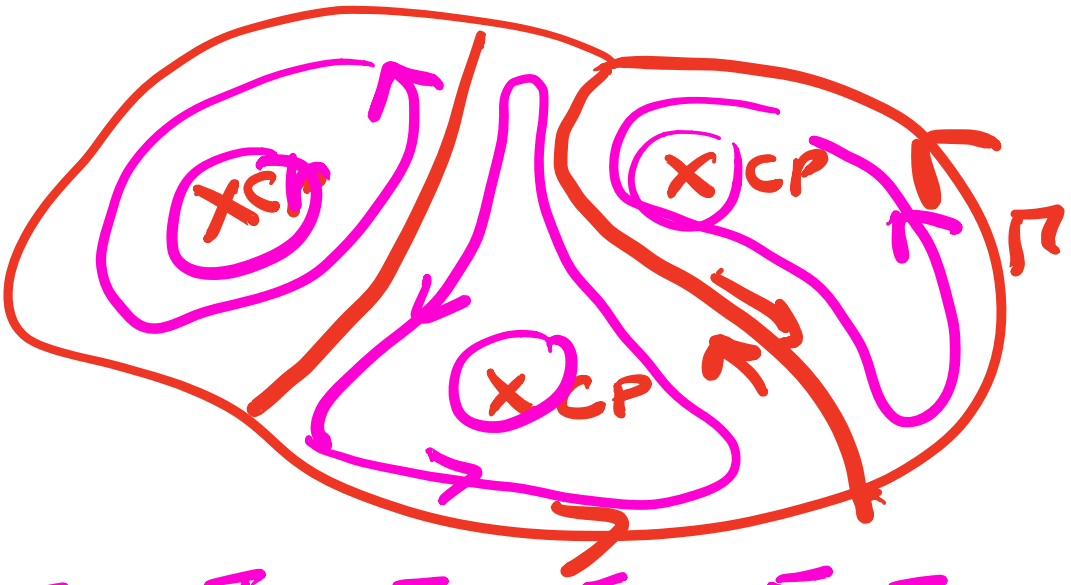
Closed Orbit



Index = 1



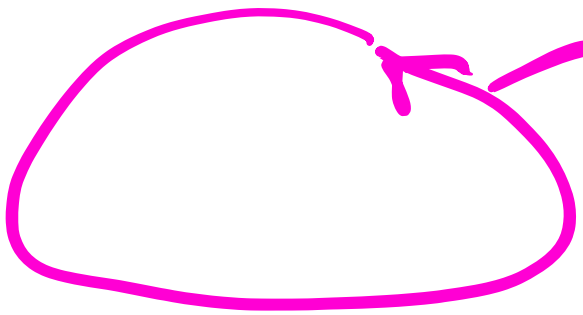
$\Gamma = \Gamma_1 + \Gamma_2$ $I_\Gamma = I_{\Gamma_1} + I_{\Gamma_2}$



$$I_\Gamma = I_{\eta_1} + I_{\eta_2} + I_{\eta_3} = \sum_{C.P.} I_n$$

Index curve =

Σ Index enclosed C.P.



Periodic orbit

Index = 1

$$V = V(x,y) = r e^{i\theta}$$

$$\theta = \theta(x,y)$$

$$\tan\theta = \frac{v_y}{v_x}$$

$$x = x(t), \quad y = y(t)$$

$$\frac{d\theta}{ds} = \frac{1}{1+\theta^2}\left(\frac{d}{ds}\left(\frac{v_y}{v_x}\right)\right)$$

$$\Delta\theta = \int_{s_0}^{s_1} \frac{d\theta}{ds} = \int$$

$$\dot{\phi} = 1 - K \sin\phi$$

$$\phi = \mu(t - t_0) + \frac{\Phi}{9}(\mu(t - t_0))$$

9 periodic