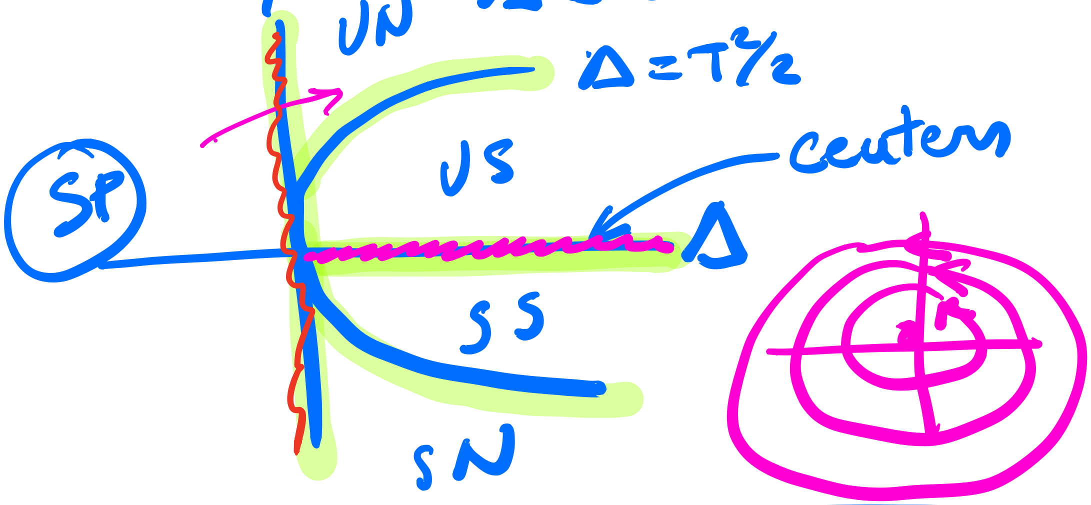
$$\dot{X} = A X \qquad A = 2\times 2 \text{ matrix}$$

$$0 = \det(A - \lambda I)$$

$$\lambda = \frac{T}{2} + \left(\frac{T^2}{4} - \Delta\right)^{1/2}$$

$$T = \text{trace } A$$

$$\Delta = \det A$$



Stability Jargon

Strong linear stable

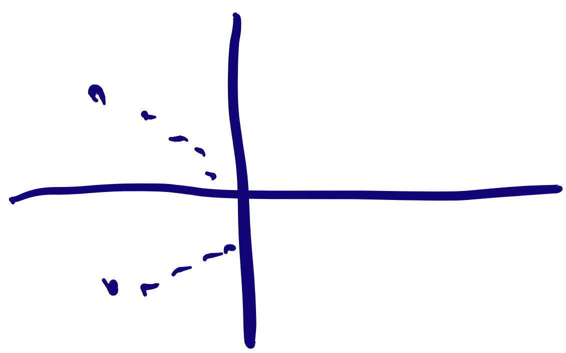
$$\mathrm{Re}(\lambda) < 0$$

For finite dim. systems

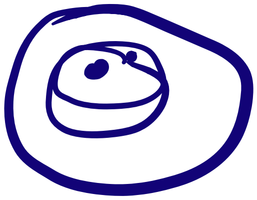
$\Longrightarrow$ Nonlinear Stab

In an ∞ dim system can have $\mathrm{Re}(\lambda) < 0$

but $\mathrm{Re}(\lambda) \to 0$

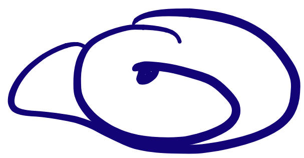


① Lyapunov stability



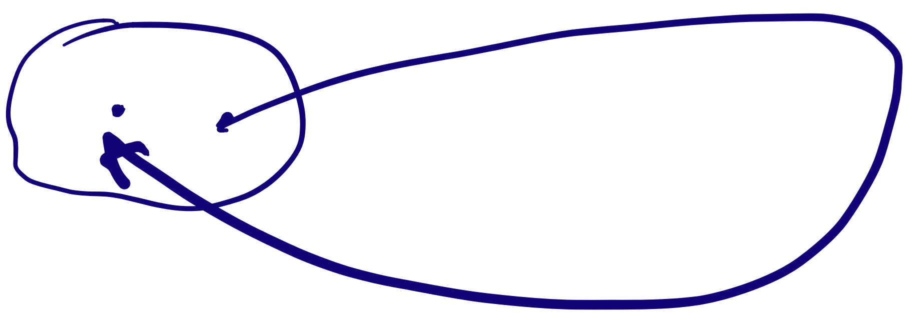
Example
centers

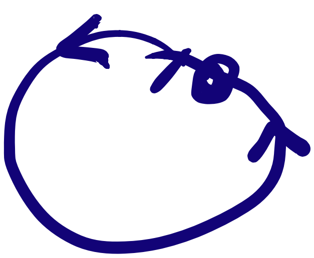
② Attractive



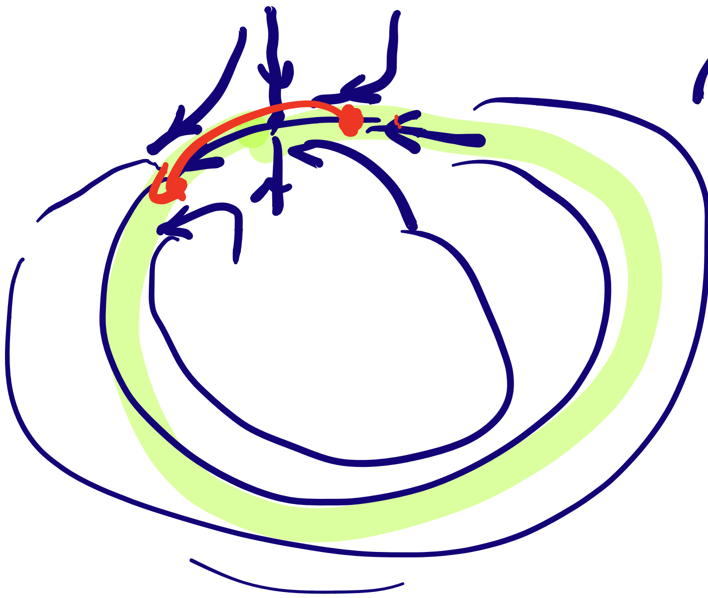
Example: Stable Nodes
Stable Spirals

③ Asymptotically stable





asymptotically
stable but not
Lyapunov



Asympt. stable
but
not Lyapunov

Example $\dot{x} = Ax$ $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\dot{x} = y$
$\dot{y} = -x$

Center $\longrightarrow$ Spiral

$$\boxed{\begin{aligned} \dot{r} &= 0 \\ \dot{\theta} &= \omega \end{aligned}}$$

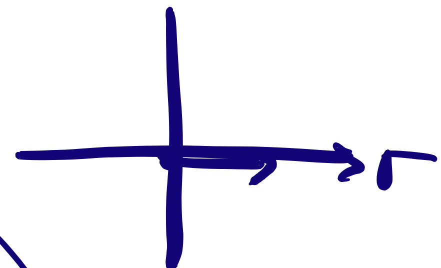
$$\begin{cases} \dot{x} = -\omega y \\ \dot{y} = \omega x \end{cases}$$

$$\ddot{x} = -\omega \dot{y} = -\omega^2 x$$

$$\boxed{\ddot{x} + \omega^2 x = 0}$$

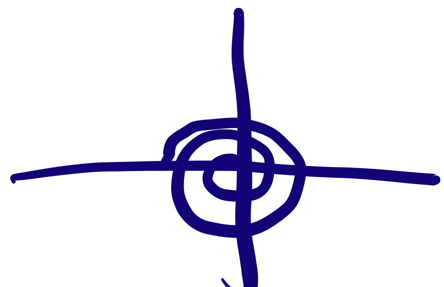
$$\boxed{\begin{aligned} \dot{r} &= f(r) \\ \dot{\theta} &= \omega \end{aligned}}$$

$\dot{r} = \pm r^3$



$$\dot{x} = -\omega y \pm r^2 x$$
$$\dot{y} = \omega x \pm r^2 y$$

$r^2 = x^2 + y^2$

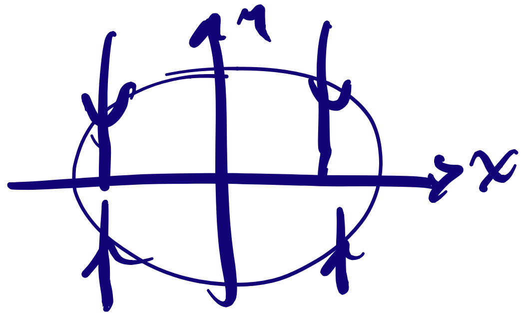


$$\boxed{\dot{x} = x^3 \qquad \dot{y} = -y}$$

$x = y = 0$

C.P.

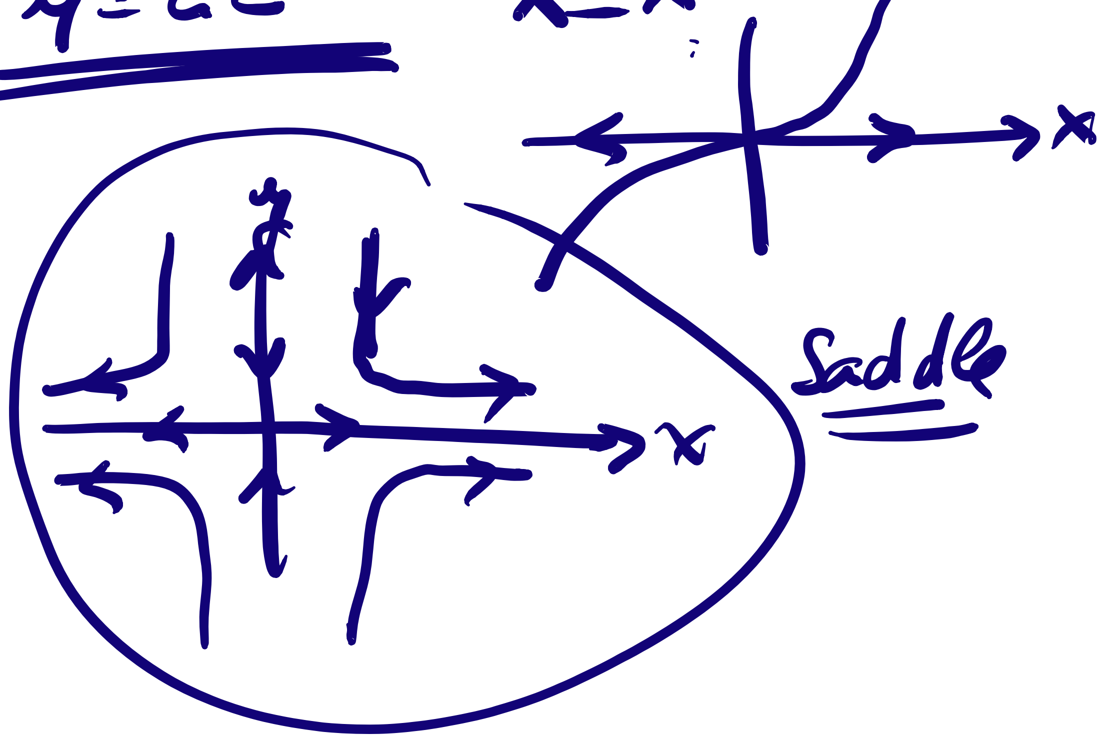
$\dot{x} = 0$
$\dot{y} = -y$ — always — only C.P.



all $y=0$ are C.P. !!

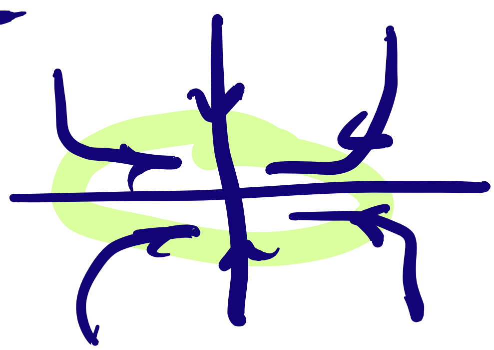
$\dot{y} = a e^{-t}$

$\dot{x} = x^3$



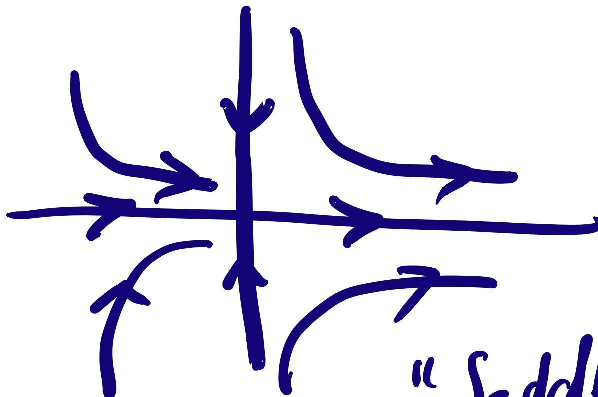
Saddle

$\dot{x} = x^3 \longrightarrow \dot{x} = -x^3$



Stable Node

$\dot{x} = x^2$



"Saddle-node"

Rabbits and sheep

Read in book

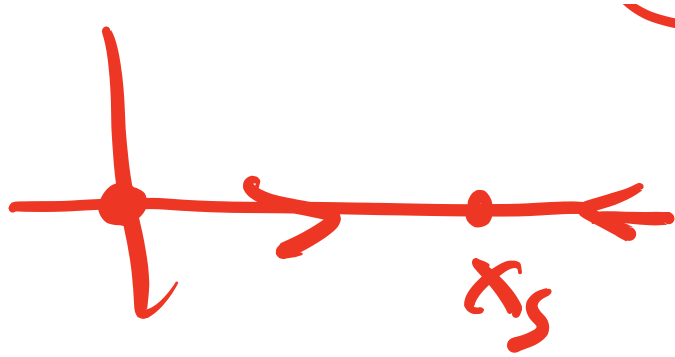
$$\boxed{\dot{x} = r x}$$

e^r

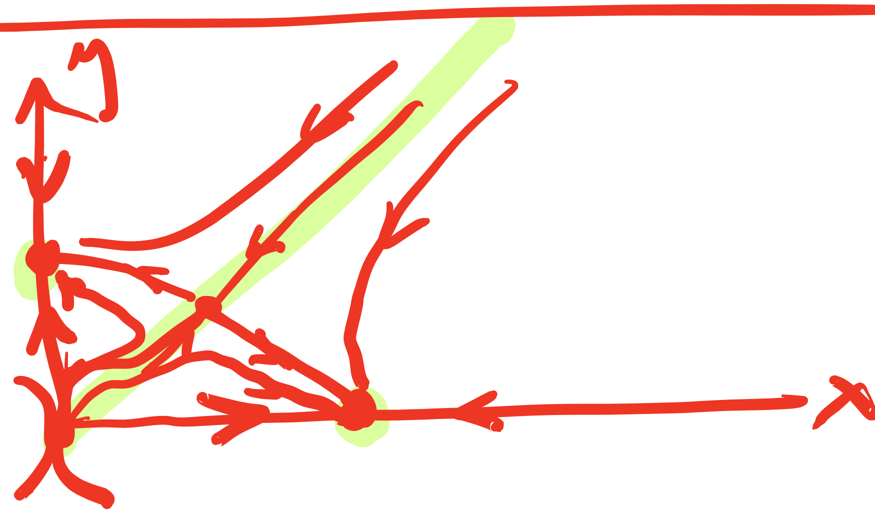
$$r = r_0\left(1 - \frac{x}{x_s}\right)$$

Logistic Model

$r_0 > 0$



All models are wrong
but some are useful



Example $m\ddot{x} = F = -\frac{\partial V}{\partial x}$

$F = -\frac{\partial V}{\partial x}$, $V = V(x)$

Mechanics no dissipation

$m\dot{x}\ddot{x} = -\frac{\partial V}{\partial x}\dot{x}$

$\frac{d}{dt}\left[\frac{1}{2}m\dot{x}^2\right] = -\frac{d}{dt}V$

$$\boxed{E = \frac{1}{2}m\dot{x}^2 + V}$$ const

Cons. Energy

Dyn System is conservative if it has a

quantity $E = E(x)$

$\dot{x} = F(x)$ $x \in$ phase space

Along orbits E is constant

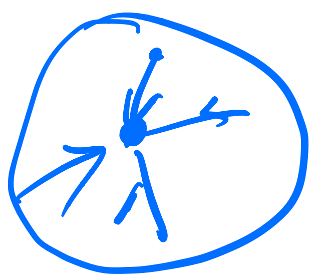
$\nabla E \neq 0$ except at C.P.

Mech system $E = \frac{1}{2} m\dot{x}^2 + V$

$\nabla E = [V_x, m\dot{x}]$

$\ddot{x} + F(x) = 0$

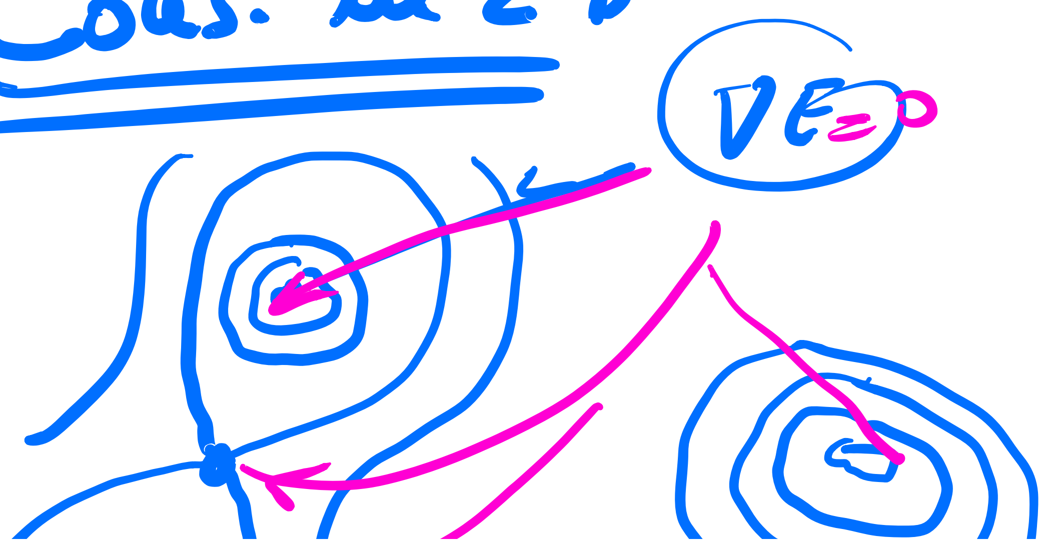
#1 Cons. systems do not have attractors.



Attractor $\Rightarrow E = \text{const}$ near C.P.

NOR Repellers

Cons. in 2-D



Centers
Stay center

Generalization for more than 2-D is Hamiltonian Systems