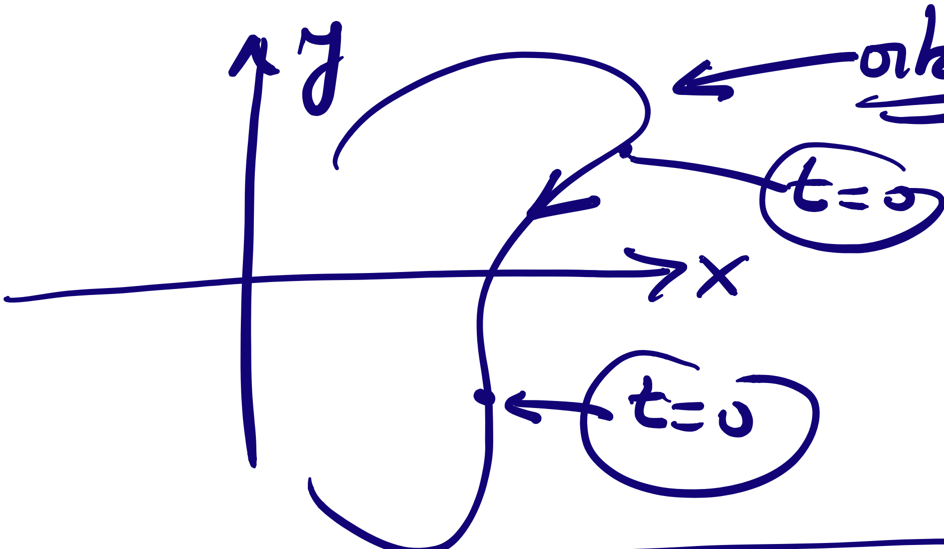
$$\boxed{\begin{aligned}\dot{x} &= f(x,y)\\ \dot{y} &= g(x,y)\end{aligned}}$$

(x,y) in **plane**

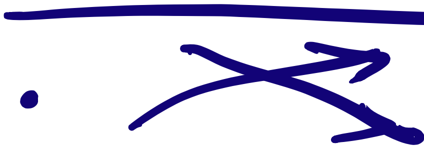


Poincare Bendixson Theorem

New from 18.03

f, g smooth (Lipschitz enough)

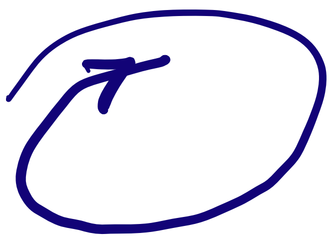
Orbits do not cross !!



(NO)!

Jordan curve lemma

Closed curve splits plane into inside and outside



Very strong restriction on orbits

One thing that Poincaré

Bendixson $\Longrightarrow$

No spaghetti

No chaos

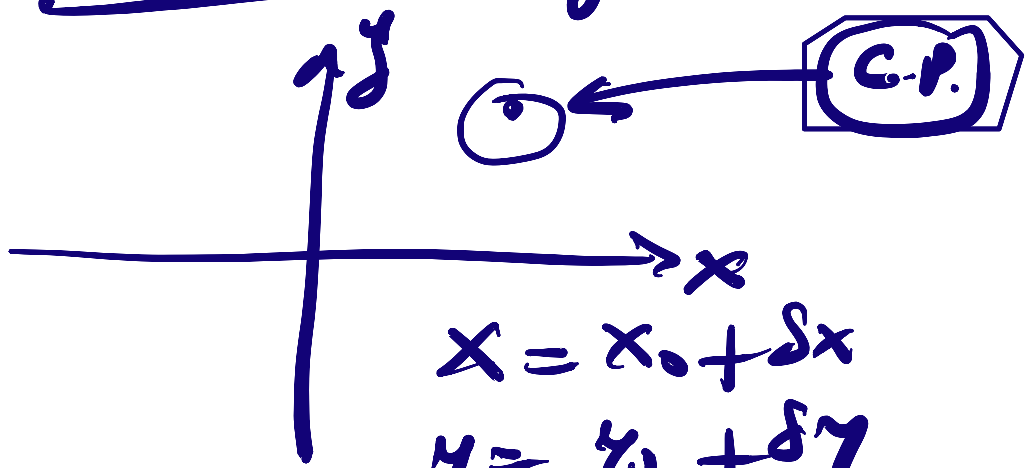
$\dot{x} = f$

$\dot{y} = g$

start from beginning

Critical points $f(x_0, y_0) = 0$

Equil. solns $g(x_0, y_0) = 0$



$$x = x_0 + \delta x$$
$$y = y_0 + \delta y$$

$$\boxed{\frac{d}{dt}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = A \begin{pmatrix} \delta x \\ \delta y \end{pmatrix}}$$

Describes syst. near C.P. "<u>sort-of</u>"

<u>Jargon</u> below

SS = structurally stable

$$\boxed{\dot{X} = AX}$$

A is 2×2 matrix const.

X = 2-vector

$$A\lambda_j = \lambda_j \vec{v} \qquad j = 1, 2.$$

$$X = a\vec{v}_1 e^{\lambda_1 t} + b\vec{v}_2 e^{\lambda_2 t}$$

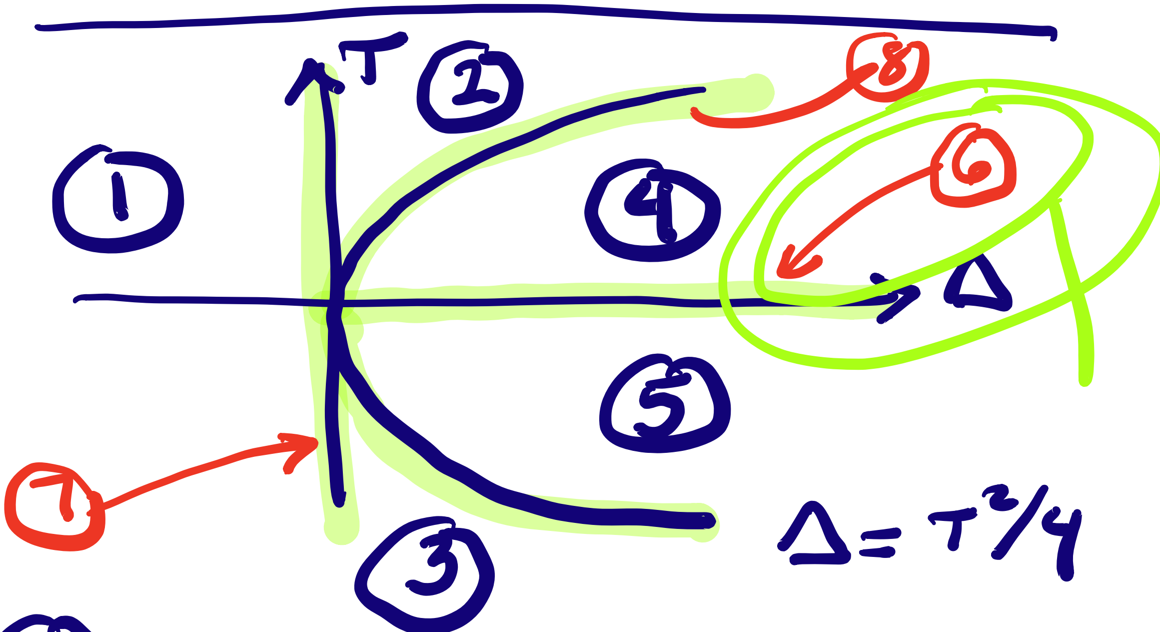
$$\det(A - \lambda I) = 0$$

$$\lambda^2 - \tau\lambda + \Delta = 0$$

$$\tau = \text{Trace}(A) = \lambda_1 + \lambda_2$$

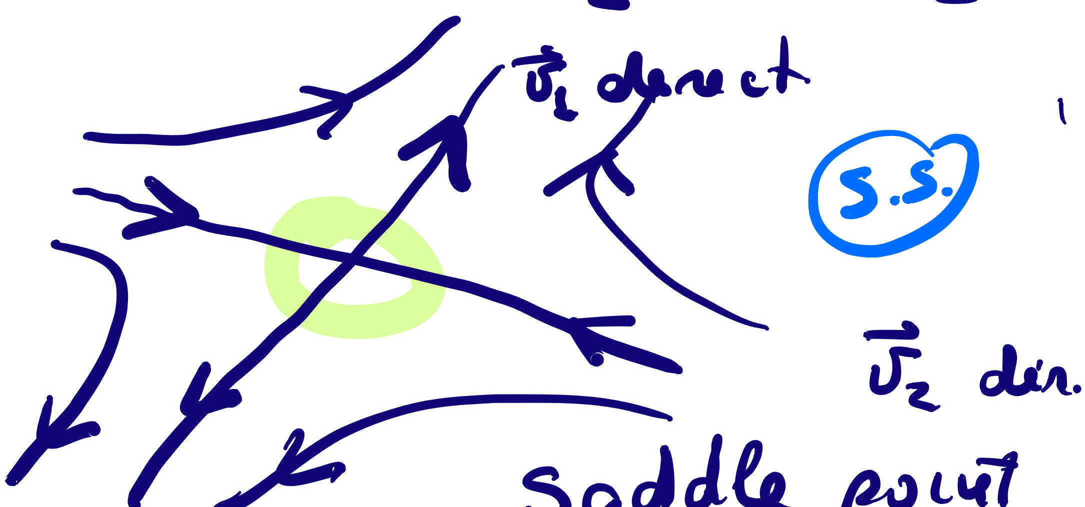
$$\Delta = \det(A) = \lambda_1 \cdot \lambda_2$$

$$\lambda_j = \frac{T}{2} + \sqrt{\frac{T^2}{4} - \Delta}$$

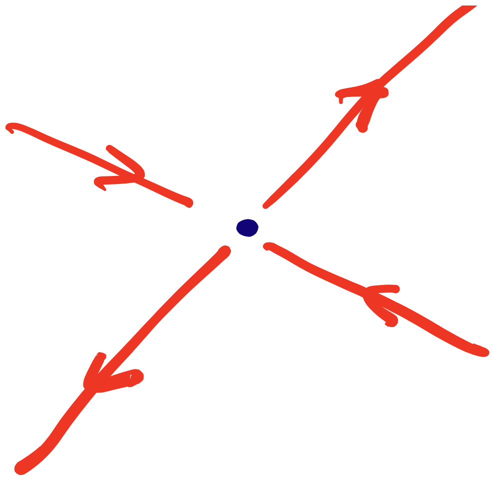


1 $\underline{\underline{\Delta < 0}}$ Two real eigenvalues

$$\lambda_2 < 0 < \lambda_1$$

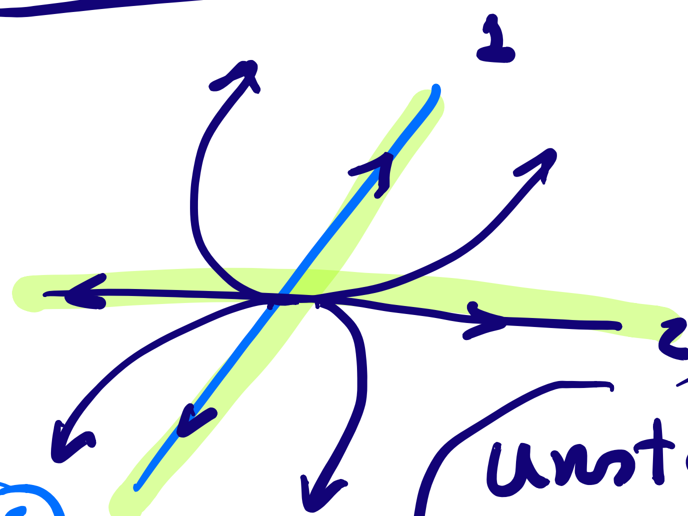


Saddle point



No crossing of orbits

② Two ev. + $0 < \lambda_2 < \lambda_1$



S.S

Unstable node

③ Flip the arrows

$\lambda_1 < \lambda_2 < 0$

stable node S.S

④ $Re(\lambda_j) > 0$ $\quad \lambda_1 = \overline{\lambda_2}$

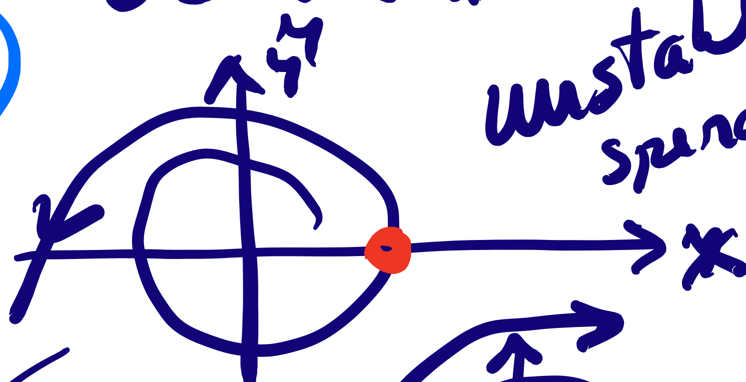
$Im(\lambda_j) \neq 0$

$$\boxed{X = Re(a v_1 e^{\lambda_1 t})}$$

$$e^{\lambda_1 t} = e^{Re(\lambda_1) t}(\cos\theta + i \sin\theta)$$

$$\theta = Im(\lambda_1) t$$

S.S.



unstable spiral

⑤ $Re(\lambda) < 0$ $\quad \lambda_1 = \overline{\lambda_2}$

S.S. $Im(\lambda) \neq 0$

Flip the arrows

te Stable spiral

⑥ $\lambda_1 = \bar{\lambda}_2$ & $Re(\lambda) = 0$

$e^{i\theta} = \cos\theta + i\sin\theta$ Euler's formula

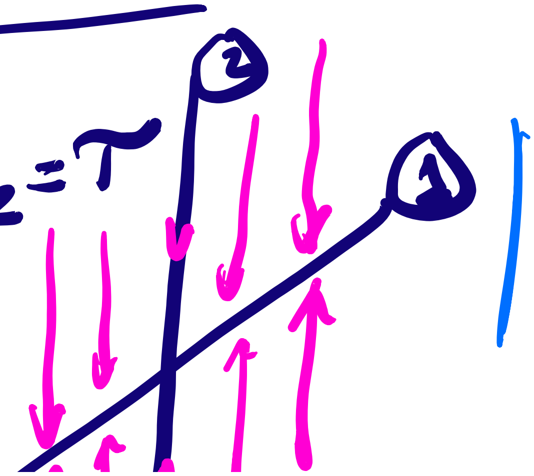
N.S.S.

Center

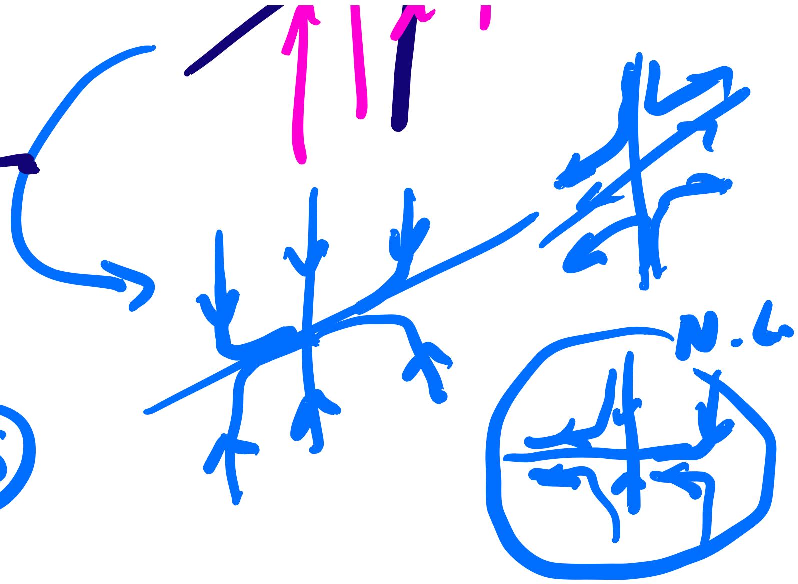
"Neutrally stable"

⑦ $\lambda_1 = 0$, $\lambda_2 = \tau$

$\tau < 0$



perturb



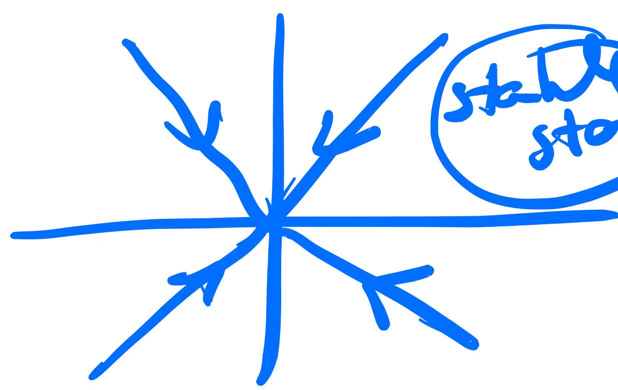
Not S.S

8 $\lambda_1 = \lambda_2 = \frac{T}{2}$ N.S.S.

Two cases There are two eigenvectors

$\boxed{A = \frac{T}{2} I}$

Star



stable star

Not S.S.

Multiplicity (geometrical)

is 1 only one eigenvector

Thus general sln

$$\boxed{X = (a\vec{v}_1 + b t\vec{v}_2)e^{\lambda t}}$$

$$A\vec{v}_1 = \lambda \vec{v}_1$$

$$(A-\lambda)\vec{v}_2 = \vec{v}_1$$ Generalized ev.

Not s.s.

Improper node