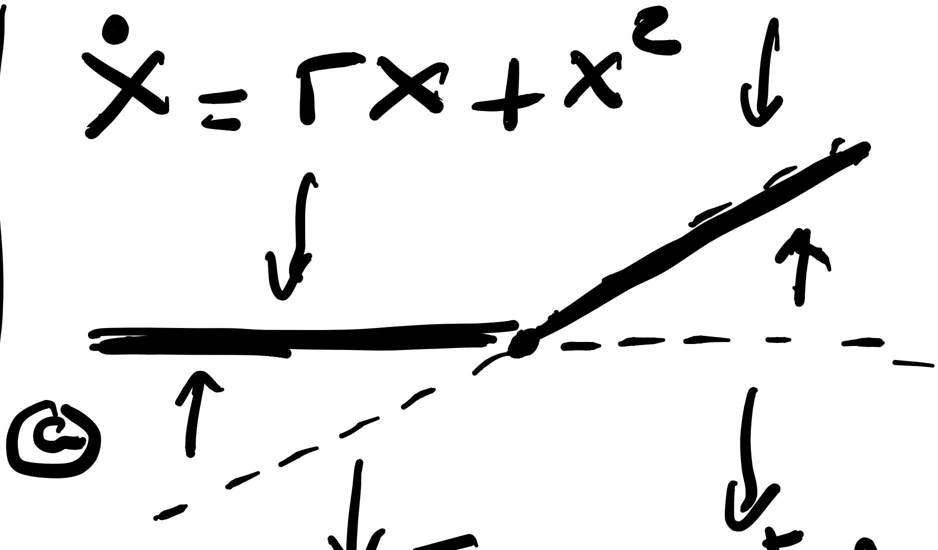
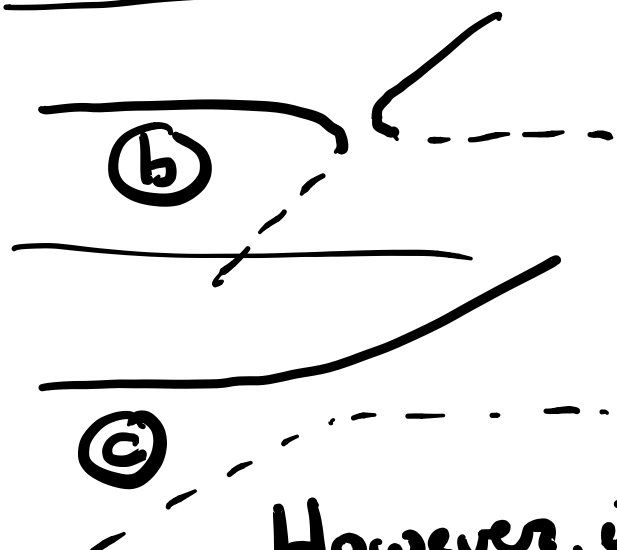
$$\dot{x} = rx + x^2$$



SS?

Transcritical

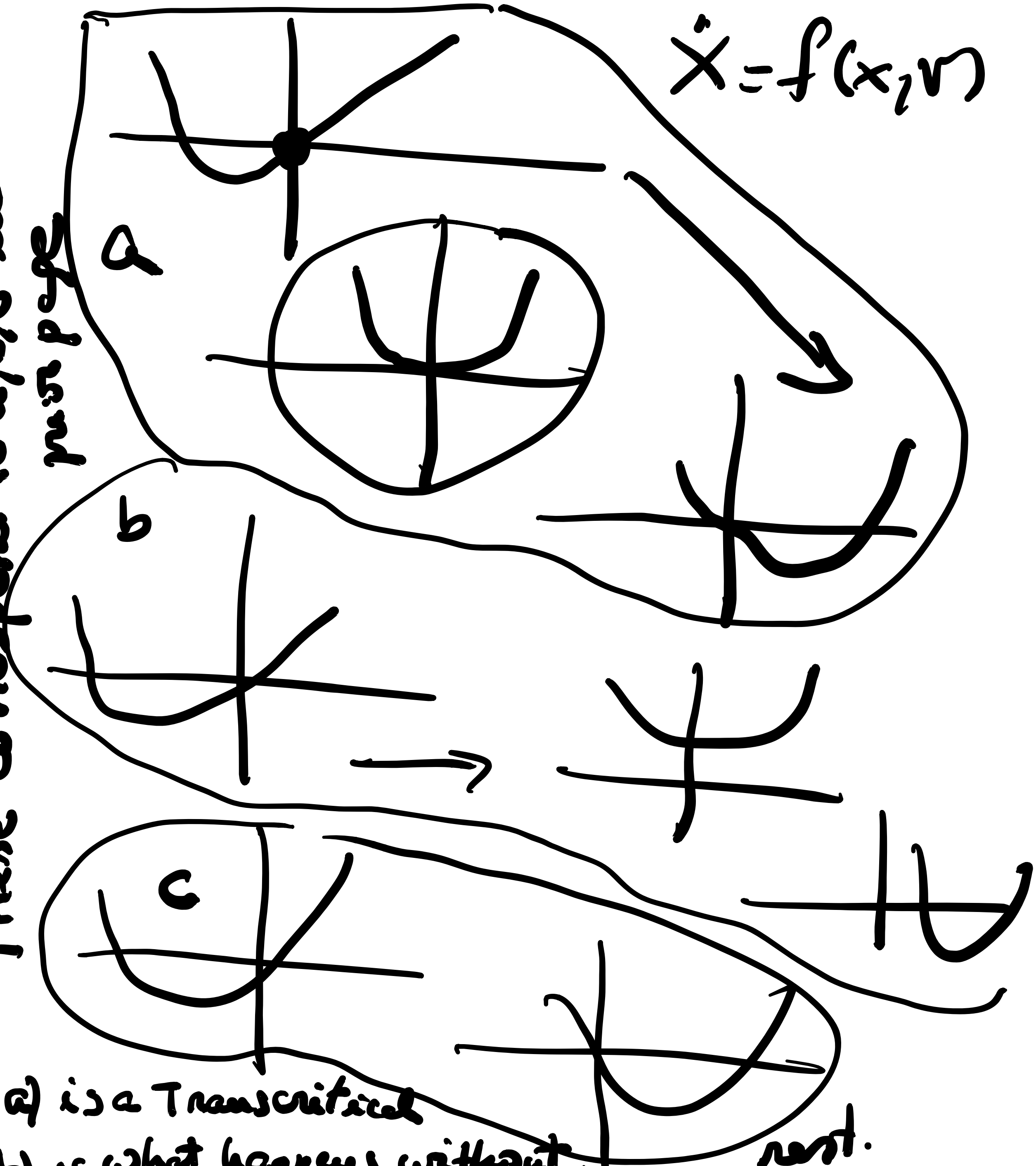


Arbitrary pert. changes bif. diagram to no SS

However, if sym. is forced to persist, then diagram cannot change $\therefore$ S.S.!

SS = Structurally Stable

$\dot{x} = f(x, r)$



These correspond to a, b, c on next page

a) is a Transcritical
b) is what happens without rest.
c) " " " " rest.

<u>Next</u>: normal forms for perturbations

$$\dot{x} = rx + x^2$$ Transcritical

standard to perturbed

$$\dot{x} = h + rx + x^2$$

Generic case. Transc. destroyed

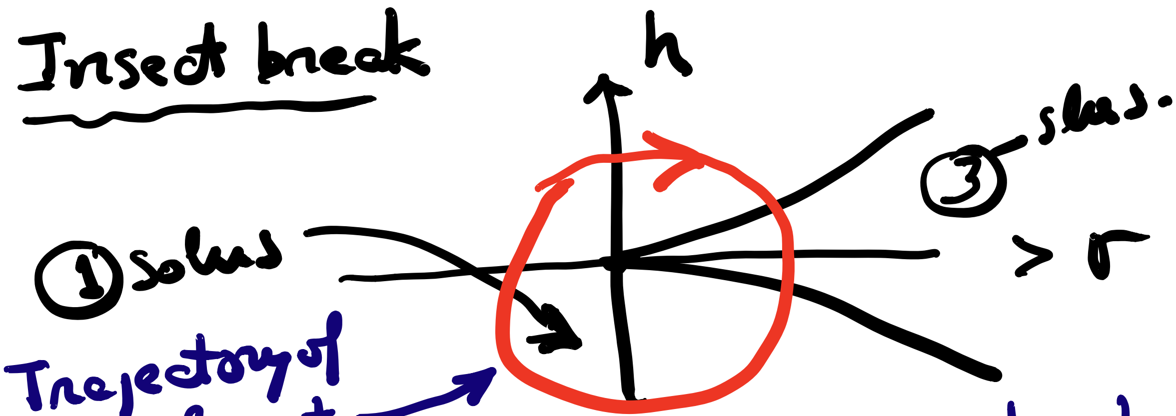
But w/ root forced to remain:

$$\dot{x} = \cancel{h_1} + (r + h_2)x + x^2$$

This cannot be here!

$\therefore$ now S.S.

Insect break



Butterfly Catastrophe

(See Insect Break pictures)

Bifurcations determined by critical points of $\dot{x} = f(x, r, h)$

i.e. slices of $\underbrace{f(x, r, h) = 0}$ ← surface

Bif. happen when $f_x(x, r, h) = 0$

∴ singular places on the surface

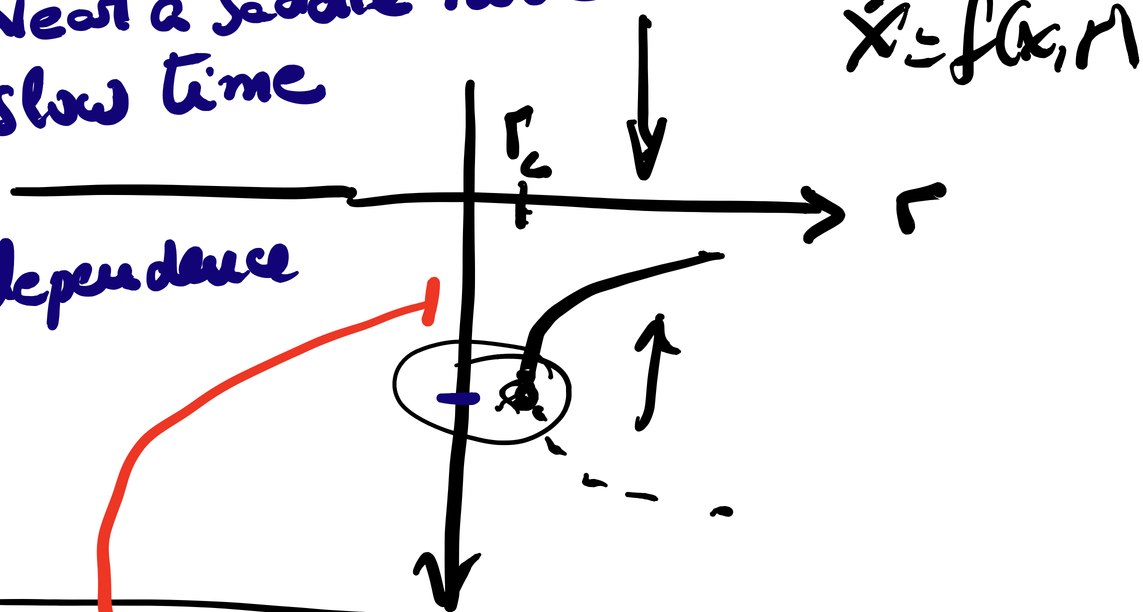
Classification of all possible singularities in low dimension (3 or 4) — R. Thom Big Theorem!

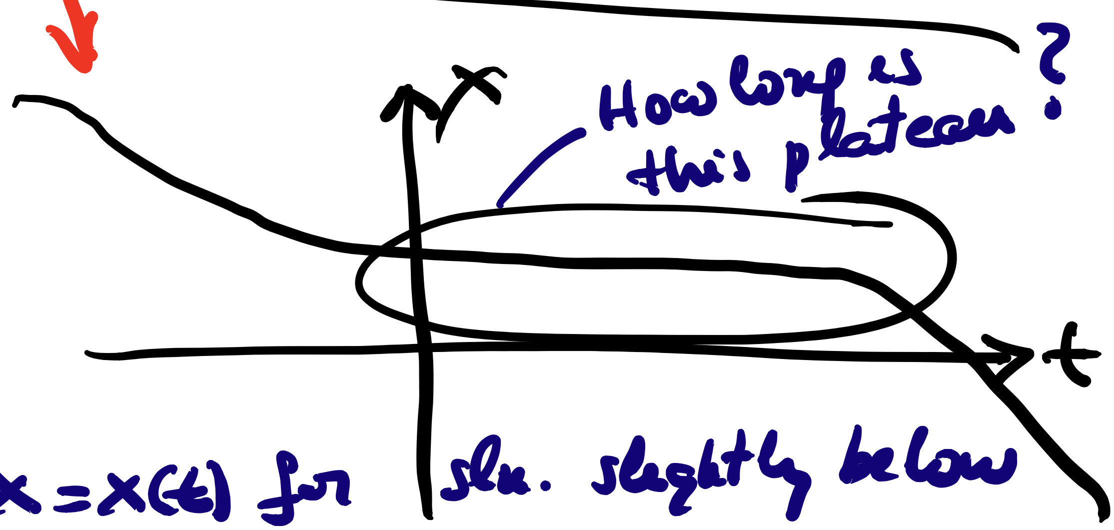
Unfortunately led to abuse.
Complicated systems do not depend on 2/3 parameters only
Huge controversy

Google H. Sussmann
Rutgers Math.

Critical slowdown

Near a saddle node slow time dependence





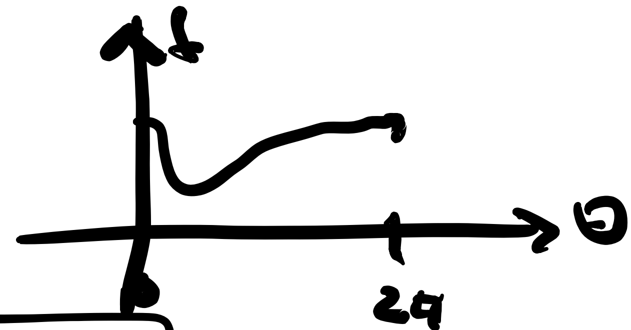
$x = x(t)$ for sln. slightly below a saddle node.

<u>Next</u>: example of critical slow down. Flow on a circle.

$$\boxed{\dot{\theta} = f(\theta, r)} \qquad \theta + 2\pi \sim \theta$$

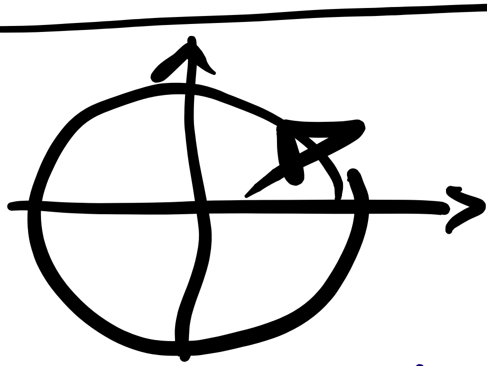
f periodic period 2π

$r > r_c$ $\quad f > 0$



$$\frac{d\theta}{f} = dt$$

$$\boxed{T = \int_0^{2\pi} \frac{d\theta}{f(\theta)}} \quad \text{period}$$

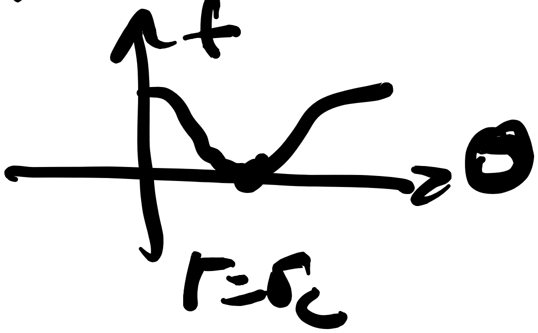


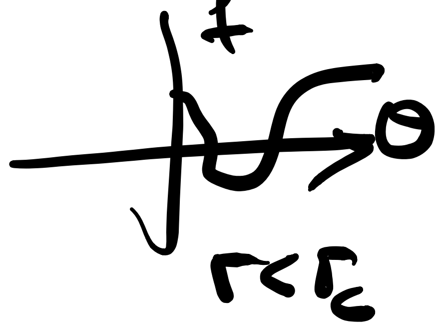
Time it takes θ to grow by 2π is T

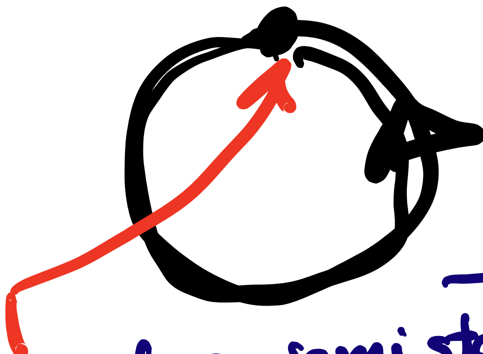
T = period

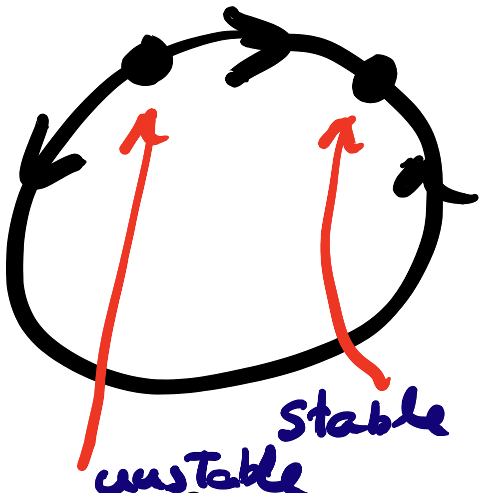
At bifurcation the periodic soln. is destroyed.

f below zero for $r < r_c$

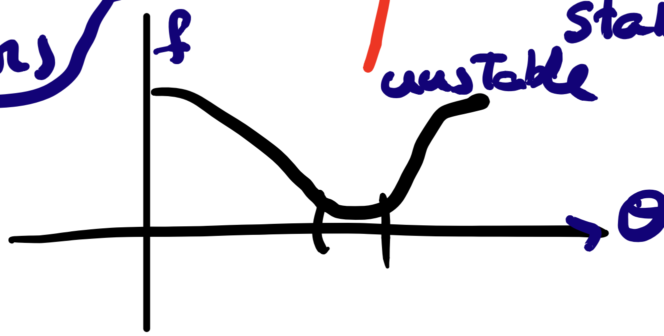


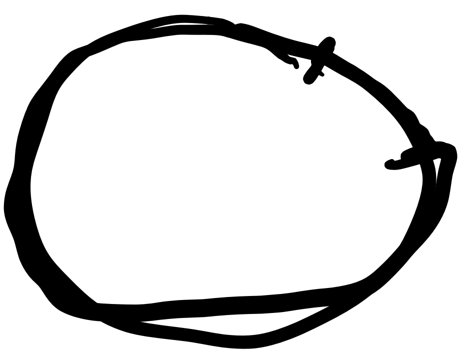


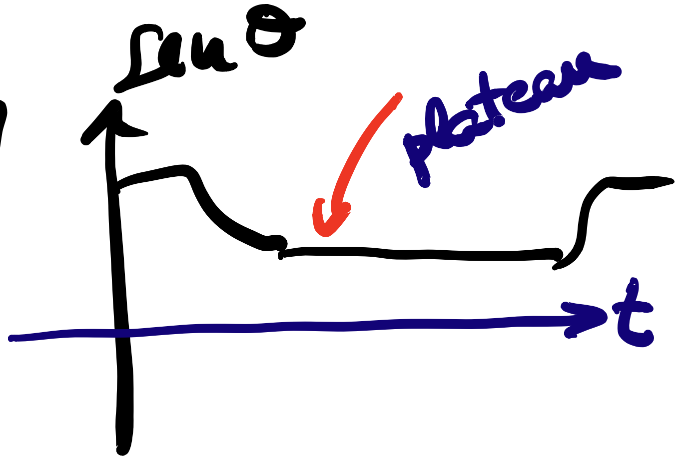




At bif. a semistable c.point appears



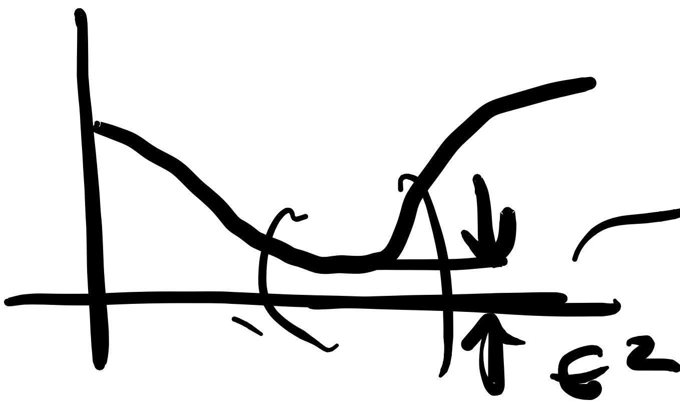




Critical slow down

$T \to \infty$ as $r \downarrow r_c$

How T $\boxed{r - r_c = \epsilon^2}$



$f \sim \epsilon^2 + a\theta^2$

$$T = \int^{2\pi} \frac{d\theta}{f} \sim \int_{-\infty}^{\infty} \frac{d\theta}{\epsilon^2 + \theta^2} \sim \frac{1}{\epsilon}$$