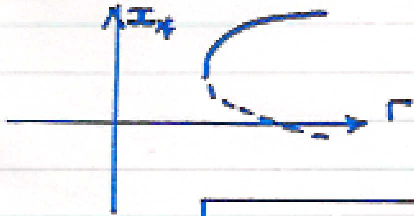
[redacted] Curves arise from sln. of some equation, and should depend cont. on parameters.

⊛ Structural Stability of bifurcations – §3.6 Imperfect Bifurcations & Catastrophes

Do some topological – hand waving arguments about it [redacted]

a) Saddle Node / Turning point

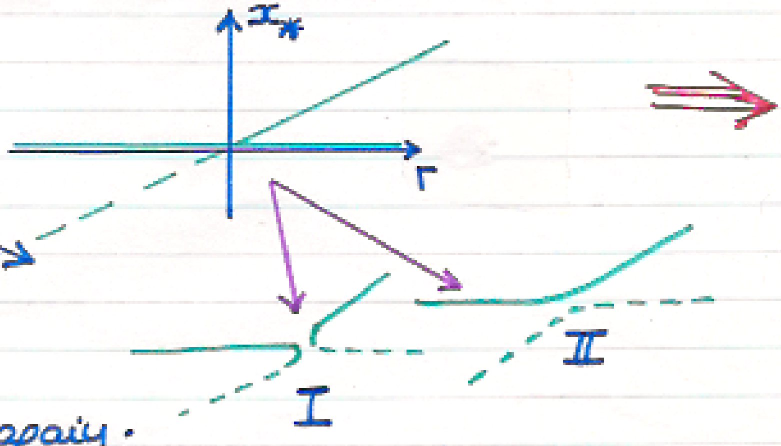


a.1) hard to destroy nose in curve!

a.2) Corresponds to a max (or min.) in $\boxed{\dot{x} = f(x, r)}$ $f(x,r)$ moving up and down through zero as r varies! An "imperfection" (slight change in f) will change the location of the nose in (x_*, r) space – when does min. hit zero – but nothing else.

Structurally Stable

b) Transcritical



b.1) Topology easy to change.

b.2) Two zeros of $f(x,r)$ collide and split up again;

---

* These show conclusions are in fact far more general than for simple first order scalar systems!

easy for an "imperfection" in f to destroy / stop collision!

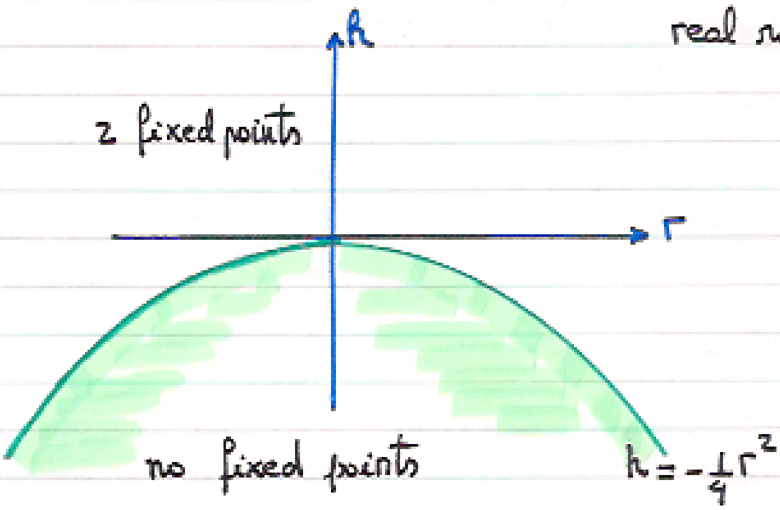
◇ Example $\dot{x} = f = h + rx - x^2$

h ← imperfection (one simple example!)

($h = 0$ yields the transcritical bif.) [No bif. for $h \neq 0$]

zeros of f are: $x_* = \frac{1}{2}r \pm \sqrt{\frac{1}{4}r^2 + h}$

real roots require $h \geq -\frac{1}{4}r^2$



"Stability" diagram

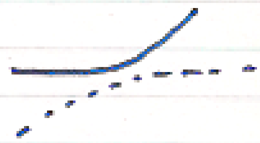
$h > 0$: Two distinct roots for all $-\infty < r < \infty$

x_*^+ is always stable

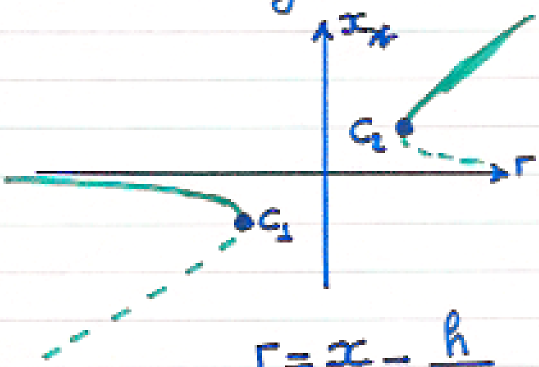
x_*^- is always unstable

$x_*^+ > x_*^-$

This is case II



$h < 0$: roots only for $|r| \geq \sqrt{-4h}$ (two if $|r| > \sqrt{-4h}$)



C_1: $x_c = -\sqrt{-h}$ $r_c = 2x_c$

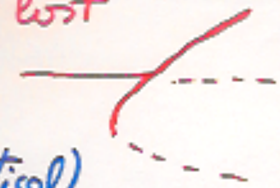
C_2: $x_c = \sqrt{-h}$ $r_c = 2x_c$

$r = x_* - \dfrac{h}{x_*}$

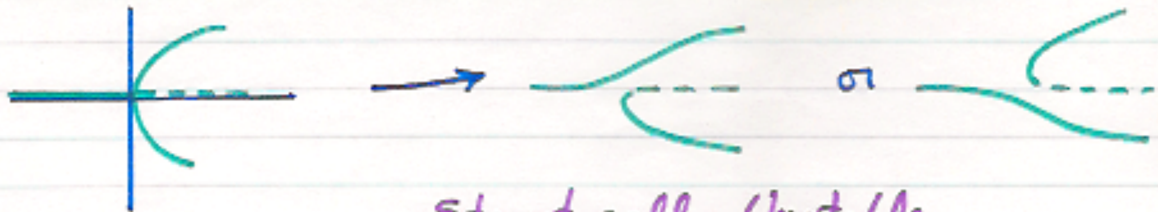
This is case I

Structurally Unstable

$\longmapsto$ or, if zero not lost



c) <u>Pitchfork</u> (supercritical – same for subcritical)

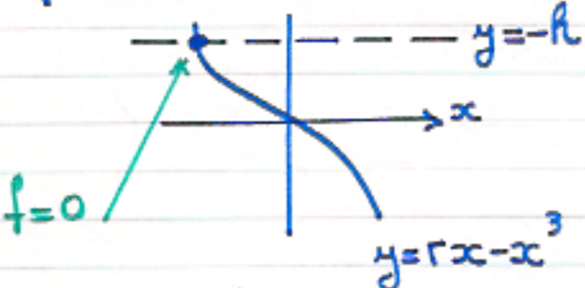


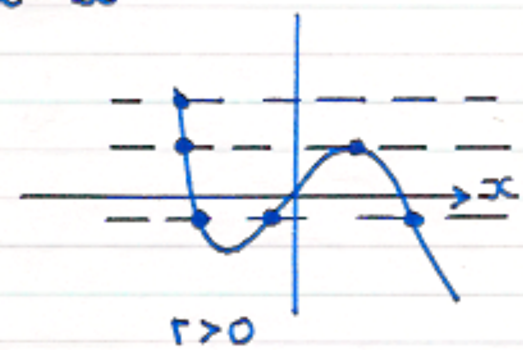
<u>Structurally Unstable</u>

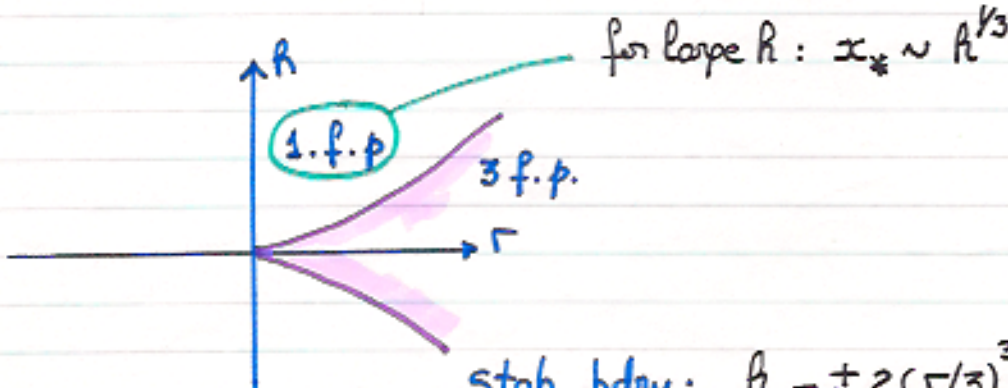
Two zeros in the complex plane hit the real axis – <u>at the same location of another zero</u> – and go real.

If they "miss" then get turning point bifurcation plus simple isolated c. point (<u>as above in figure</u>)

◊ <u>Example</u> $\dot{x} = f = h + rx - x^3$







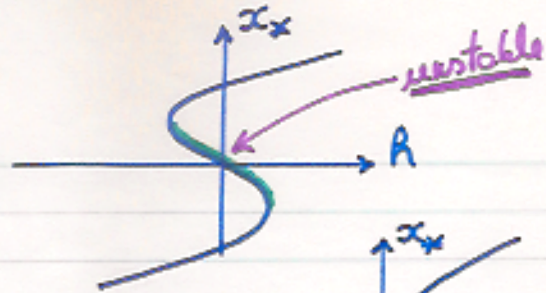
<u>Stab. bdry</u>: $h = \pm 2(r/3)^{3/2}$, $r > 0$

<u>stability diagram</u> $f = f' = 0$ (double roots)

$$\therefore \left.\begin{array}{l} 0 = h + rx - x^3 \\ 0 = r - 3x^2 \end{array}\right\} \begin{array}{l} r = 3x^2 \\ h = -2x^3 \end{array}$$

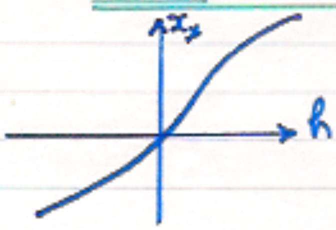
Graphs of roots for

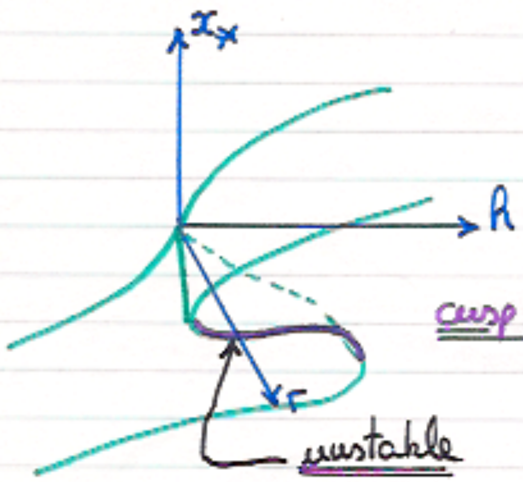
r fixed > 0 →



$r = 0$ →

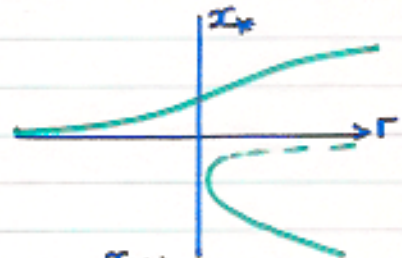
r fixed < 0 →





cusp catastrophe

h fixed > 0



h fixed < 0

catastrophes, hysteresis, etc

Example Model for Insect outbreak § 3.7

(Read in book)