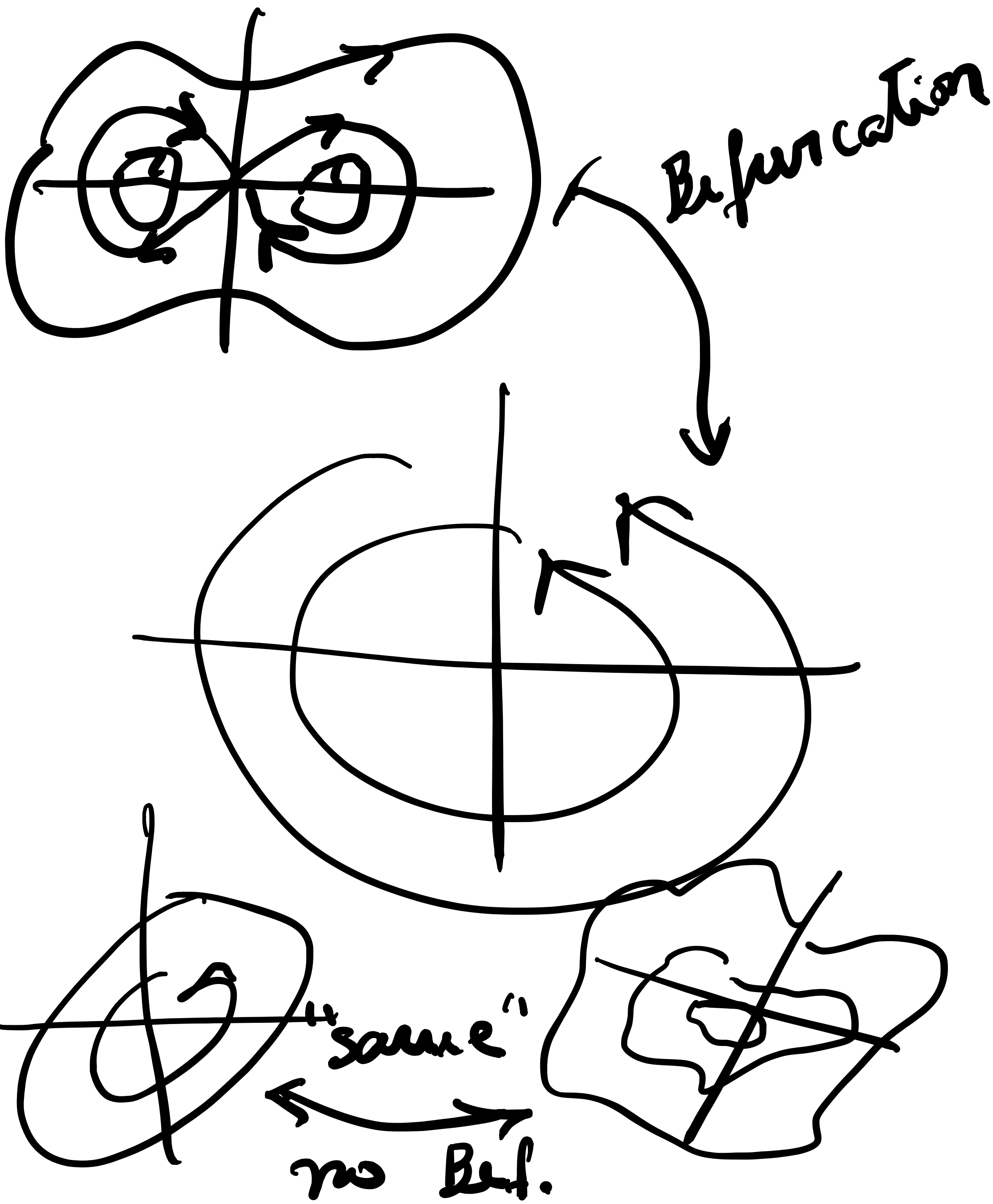
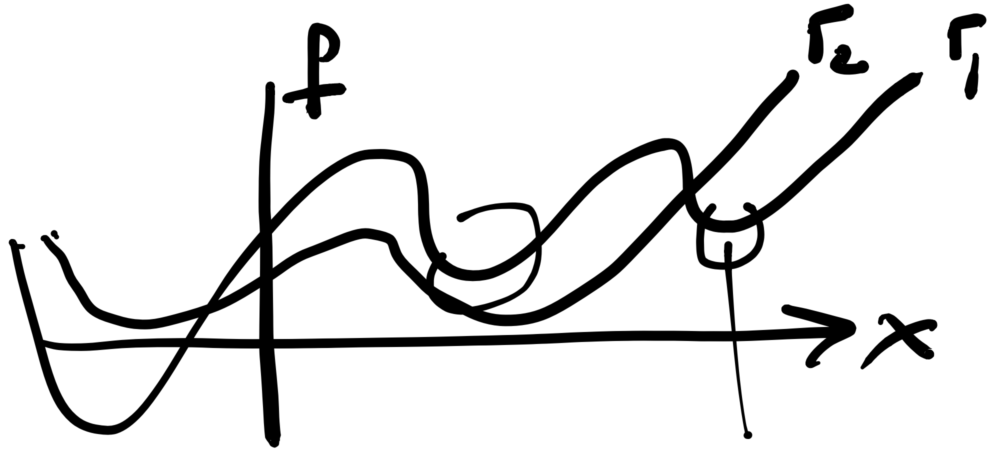
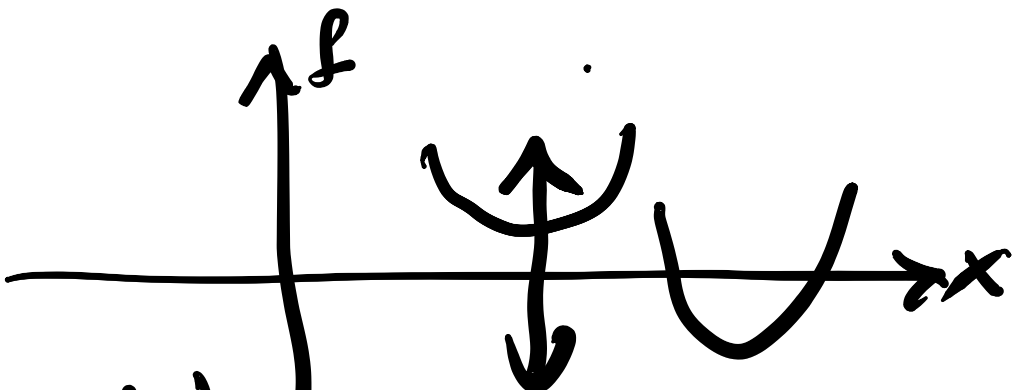
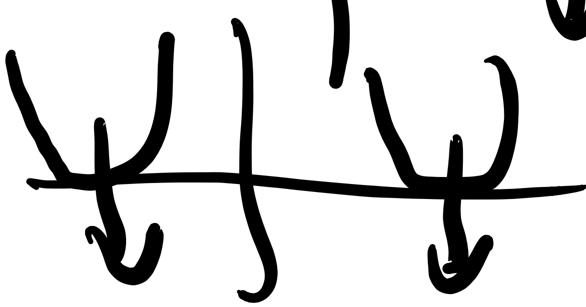
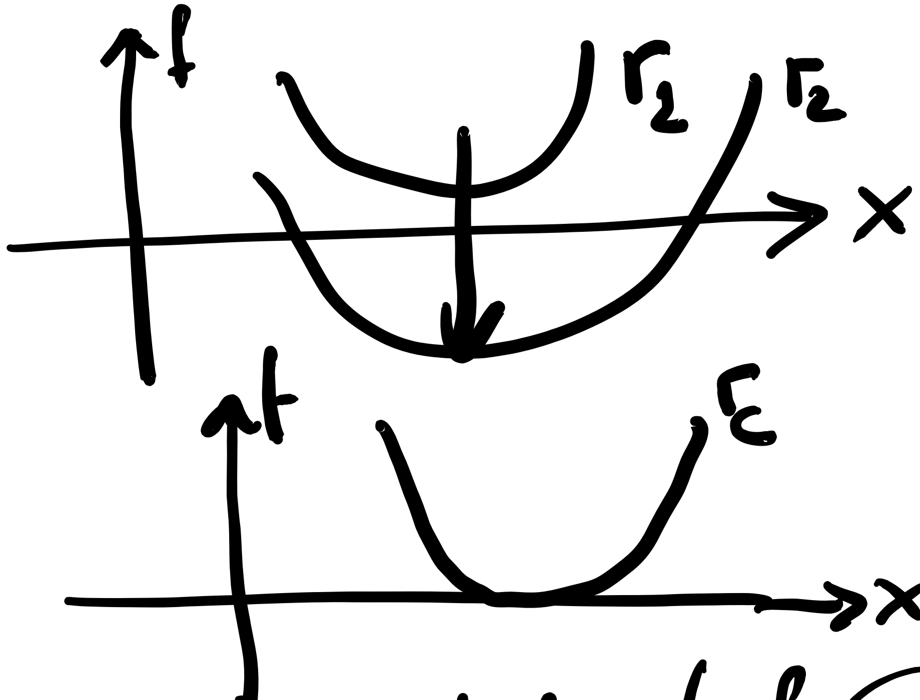

Bifurcation
"same"
no Bif.

$$\dot{x} = f(x, r)$$







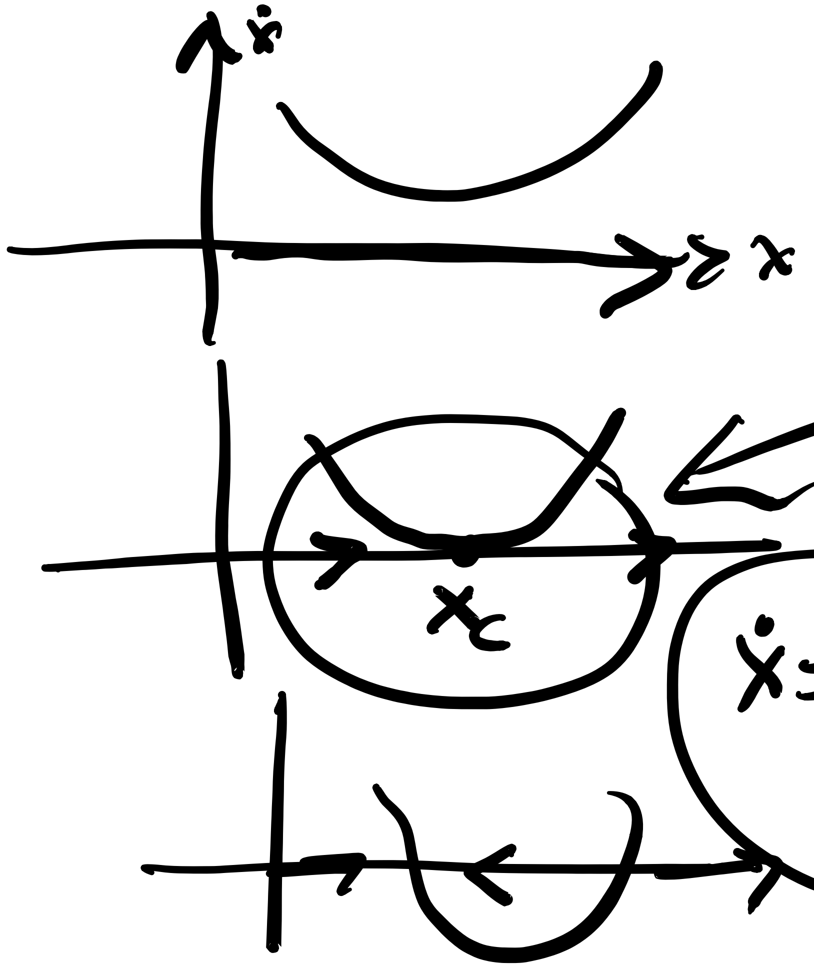


Sadde-Node bif. 2-D

turning point. bif. oo lect fwd

Fold bif. ← 2 lect. fwd

blue sky bif.



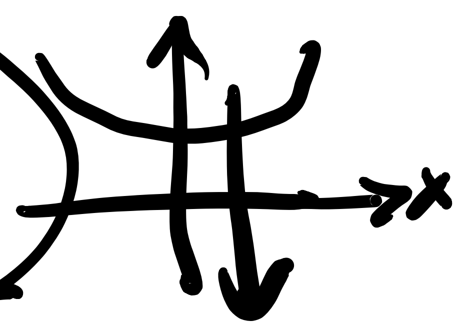
r_c

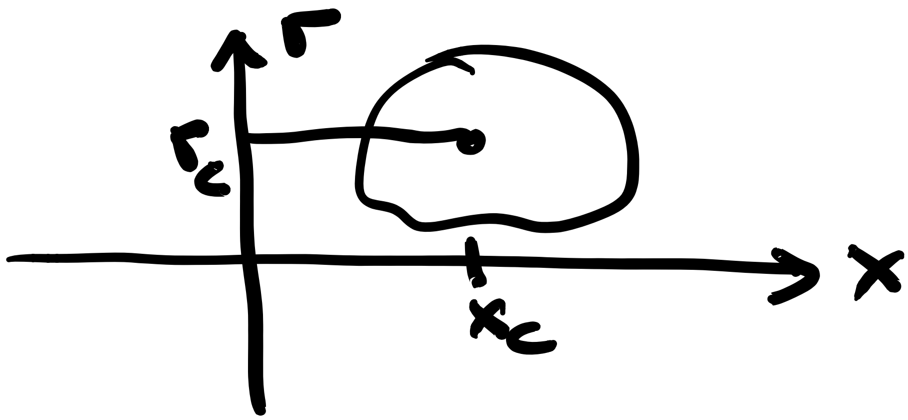
$\dot{x} = f(x, r)$

$r = r_c$

Normal form / Canonical

$$\boxed{\dot{x} = r + x^2}$$

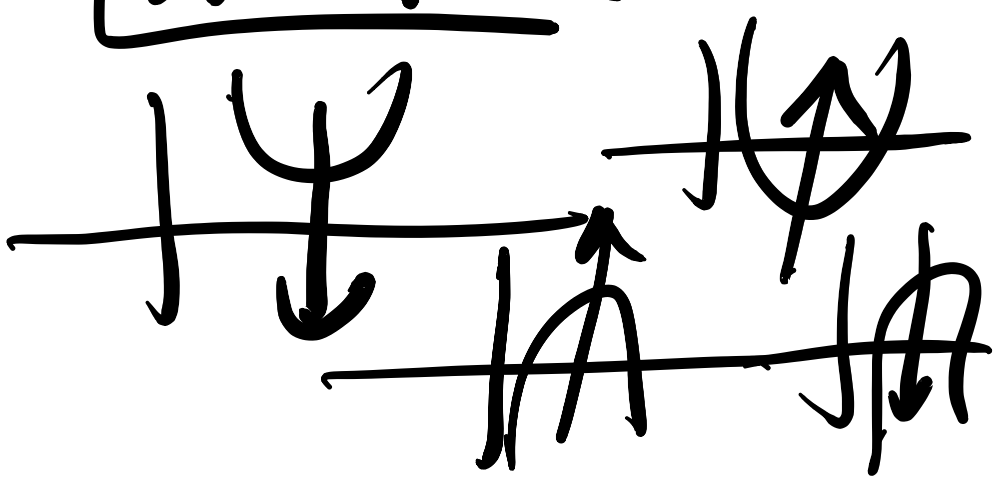


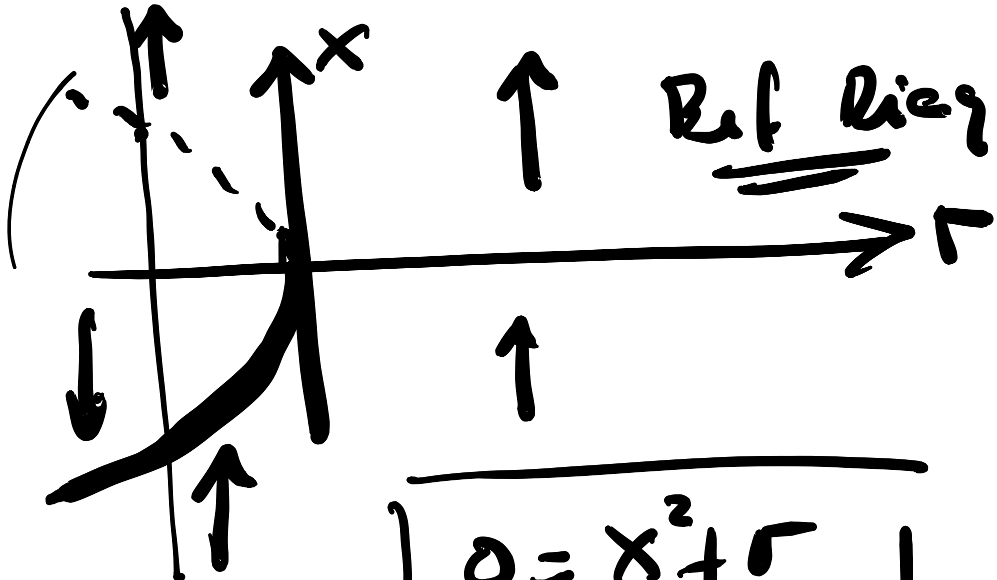


Keller Bif. Theory

$$\boxed{\dot{x} = \pm r \pm x^2}$$

$$\boxed{\dot{x} = r + x^2}$$





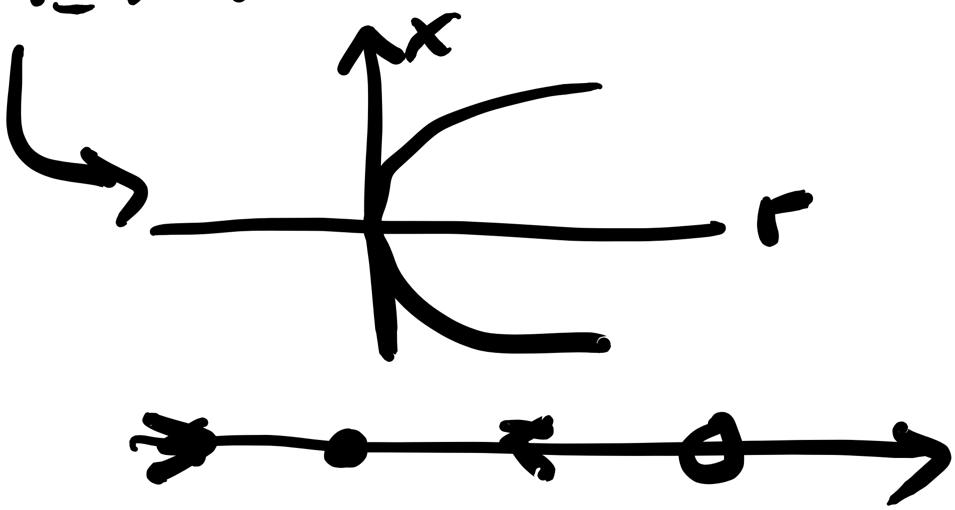
$\boxed{0 = x^2 + r}$

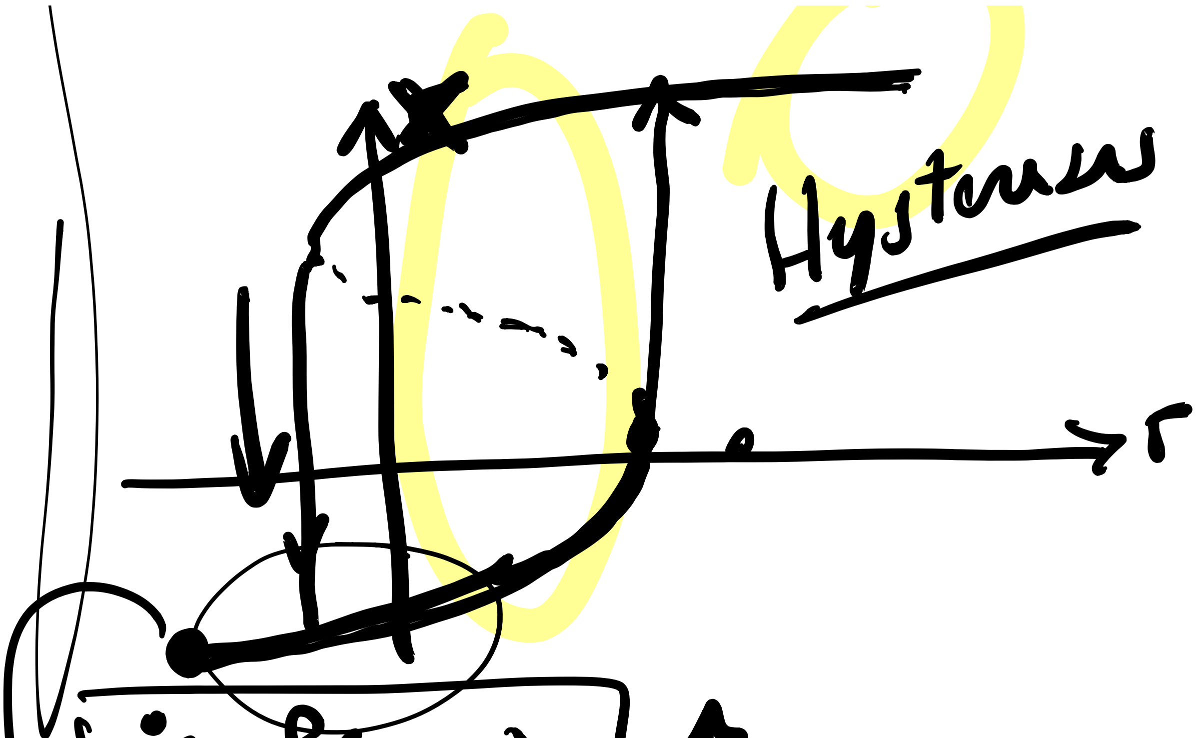
$\boxed{\dot{x} = x^2 + r}$

For $r > 0$ No. CP

For $r < 0$ $x = \pm\sqrt{-r}$

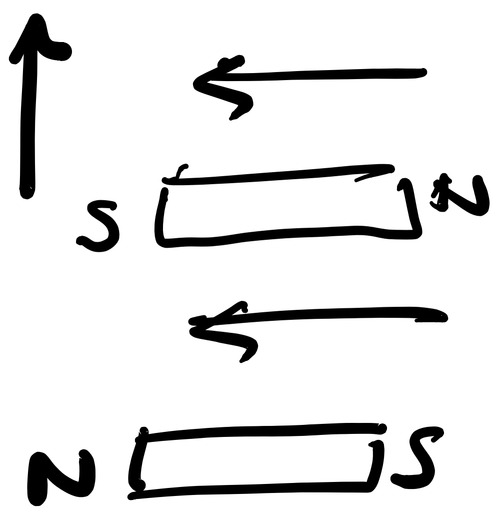
$\dot{x} = x^2 - r$



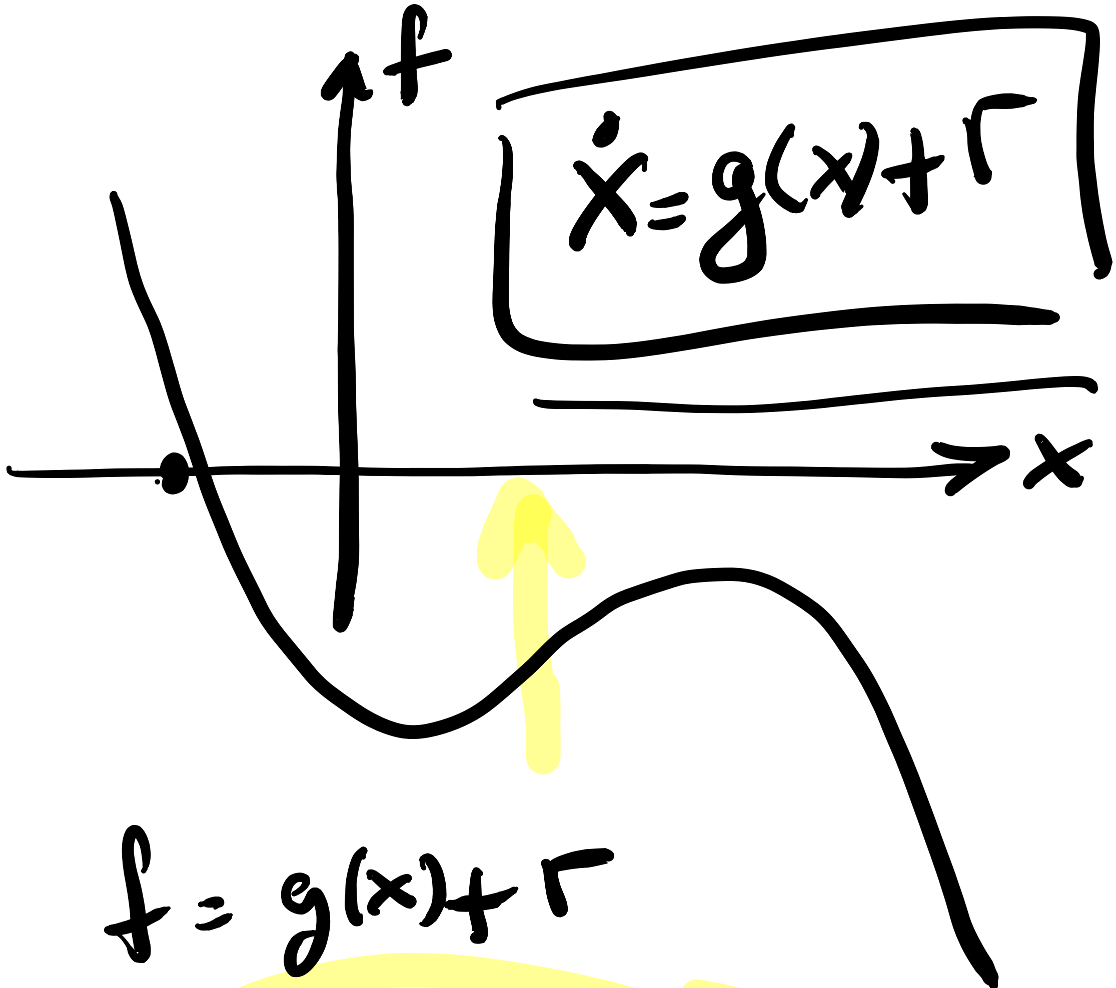


$$\boxed{\dot{x}=f(x,r)}$$

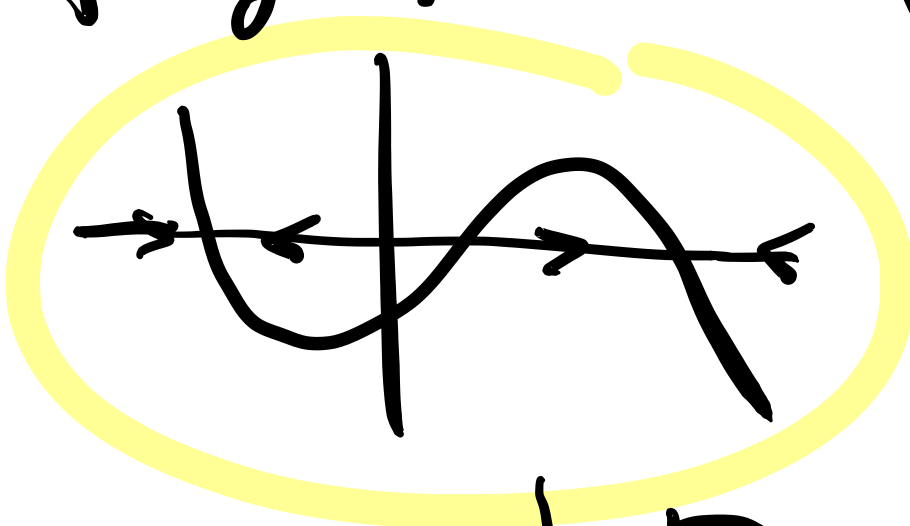
$$f(x,r)=0$$



$$\boxed{\dot{x} = g(x) + r}$$



$$f = g(x) + r$$

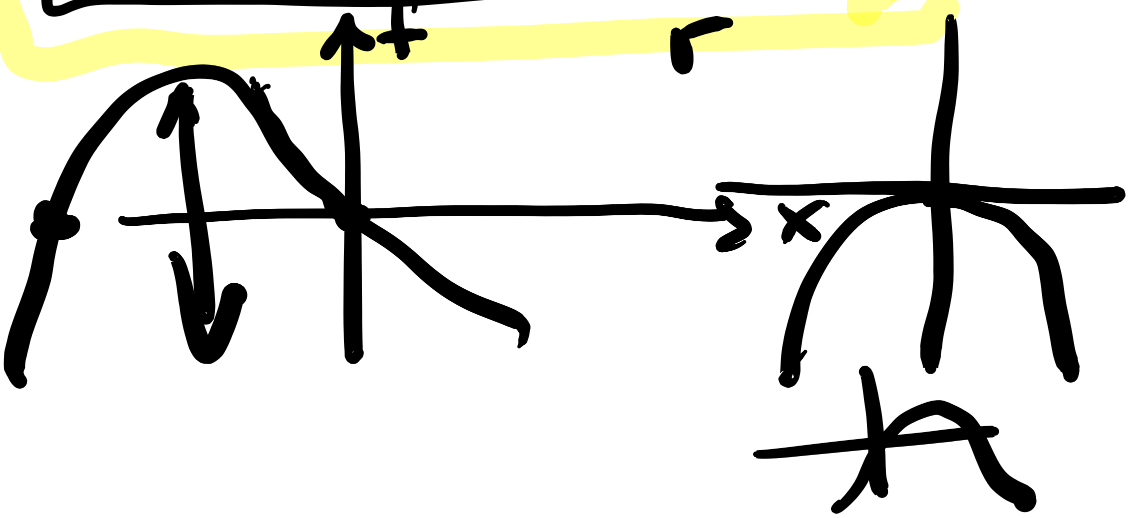


Next Add condition,

There is a c.p. that **does not** disappear!

$$\boxed{\dot{x} = rx + x^2}$$

$f_x = r + 2x$



$$\dot{x} = f(x, r)$$

with bif. at $x = x_c, r = r_c$

$$f(x_c, r_c) = 0$$

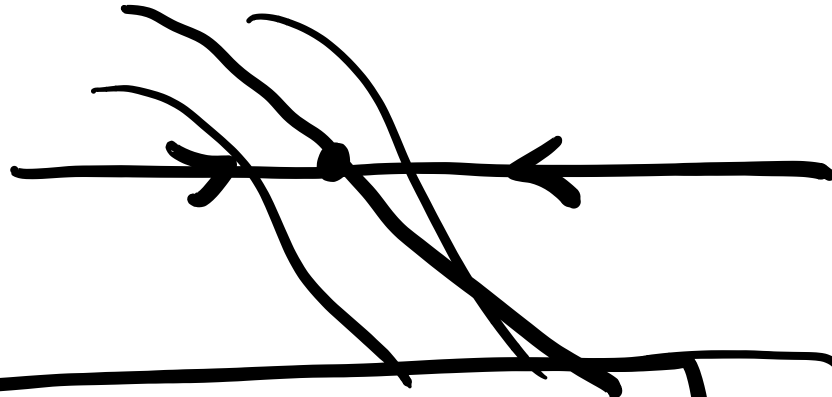
$f(x, r)$ $\boxed{f'_x(x_c, r_c) \neq 0}$

$f(x, r) = 0$

$x = x_c + \delta x$

$r = r_c + \delta r$

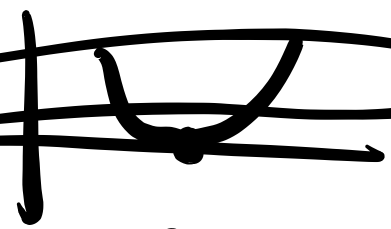
$$0 = f_x(x_c, r_c)\,\delta x + f_r(x_c, r_c)\,\delta r$$



$$\boxed{f_x(x_c, r_c) = 0}$$

Cond. For bif.

$$f(x_c, r_c) = 0$$

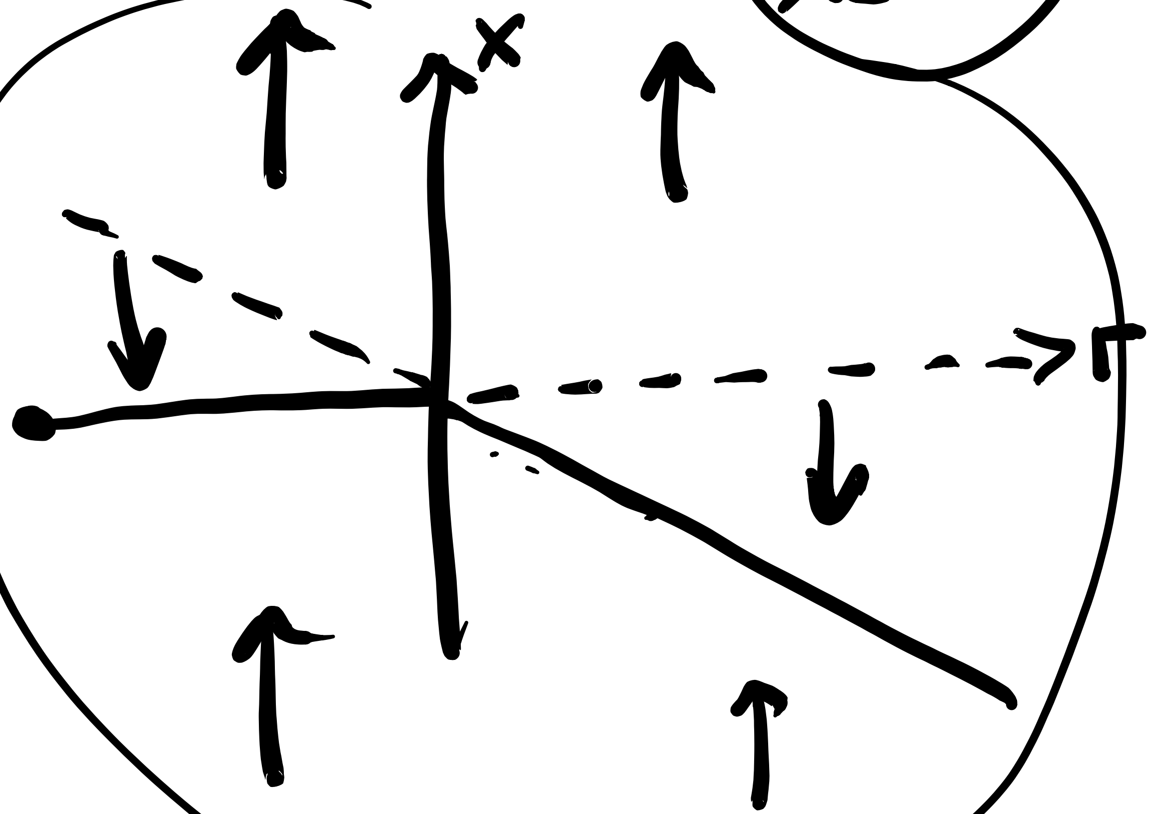


$$\boxed{f(x_c, r) \equiv 0}$$

$$\dot{x} = rx + x^2$$

$x = 0$

$x = -r$



$$f(x, r) = 0$$

$$f_x(x, r) \neq 0$$

$f(x_c, r_c) = 0$

$f_x(x_c, r_c) = 0$

$f_r(x_c, r_c) \neq 0$

$f($

$$\dot{Y} = F(Y, r)$$

$F(Y_c, r_c) = 0$

$\boxed{F_Y(Y_c, r_c)}$ not singular

no bif.