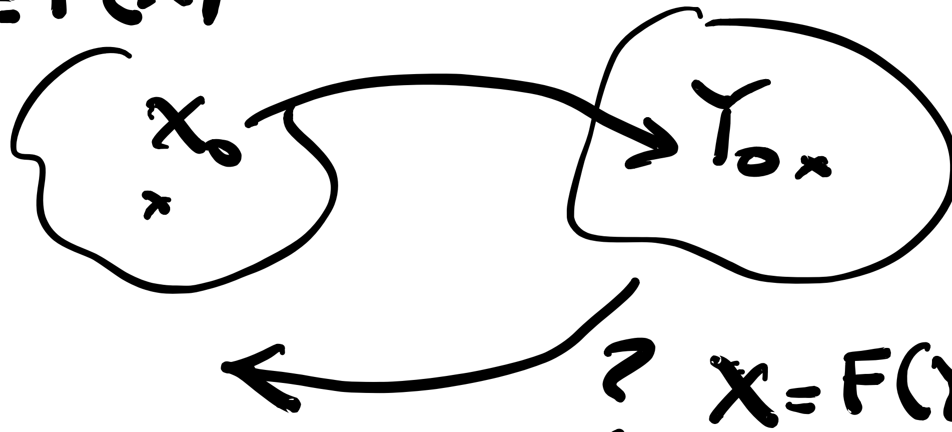
# Inverse Funct.

$Y = F(X)$




? $X = F(Y)$

Condition $\boxed{DF}$ not singular

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# Implicit Funct.

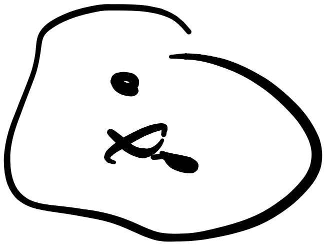
$0 = F(X, Y)$ and $F(X_0, Y_0) = 0$

$X$ $\boxed{Y = Y(X)}$ $Y$

$F(X,Y)=0 \qquad \underline{Y=G(X)}$

$F(X_0,Y_0)=0$

near $X_0$




$$\boxed{\begin{aligned} X &= X_0+\delta X \\ Y &= Y_0+\delta Y \end{aligned}}$$

$$0 = \left(\frac{\partial F}{\partial X}\right)_0 \delta X + \underbrace{\left(\frac{\partial F}{\partial Y}\right)_0}_{\text{should be square nonsing. matrix}} \delta Y$$

$$\boxed{\delta Y \approx -\left(\frac{\partial F}{\partial Y}\right)_0^{-1}\left(\frac{\partial F}{\partial X}\right)_0 \delta X}$$

$$Y = F(X) \qquad Y_0 = F(X_0)$$

$$X = G(Y) \text{ for } X \text{ close } X_0$$

$$Y = Y_0 + \delta Y \qquad X = X_0 + \delta X$$

$$\boxed{\delta Y = \left(\frac{\partial F}{\partial X}\right)_0 \delta X}$$

should be square and invertible.

$$\boxed{\delta X = \left(\frac{\partial F}{\partial X}\right)_0^{-1} \delta Y}$$

$$\boxed{\begin{aligned}\dot{Y} &= F(Y)\\ Y(0) &= Y_0\end{aligned}}$$

$$\underline{Y = Y_0 + \delta Y}$$

$$\delta\dot{Y} = \left(\frac{\partial F}{\partial Y}\right)_0 \delta Y$$

$$F(Y_0) +$$

__Metatheorem__

If linearized version of problem is structurally stable, then Nonl. Problem looks __linear one__

$\dot{Y} = F(Y)$ $\quad Y$ is $n$-vector

$\boxed{F(Y_0) = 0}$ $\quad \boxed{Y = Y_0 + \delta Y}$

$$\boxed{\delta \dot{Y} = \left(\frac{\partial F}{\partial Y}\right)_{Y_0} \delta Y}$$

Beam Equn

Beam Equn

$$u_{tt} - \gamma_1 u_{xx} + \gamma_2 u_{xxxx} = 0$$

Keller & etc.

Bifurcation Theory

Sattinger Bif. Theory

$$\boxed{\dot{x} = f(x)}$$

$$\frac{dx}{f} = dt$$

$$t - t_0 = \int_{x_0}^{x} \frac{ds}{f(s)}$$

$x = x(t)$

$x(t_0) = x_0$

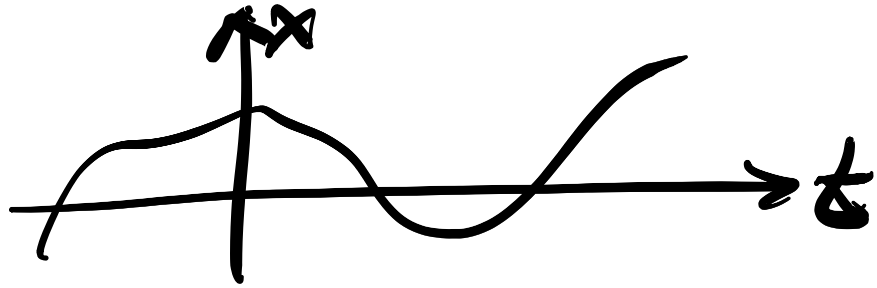
$$\boxed{\dot{x} = \sin x}$$

$$\frac{dx}{\sin x}$$

$$\frac{d}{dx} \ln\left[\frac{\sin x}{1+\cos x}\right] = \frac{1}{\sin x}$$

$x(0) = x_0$

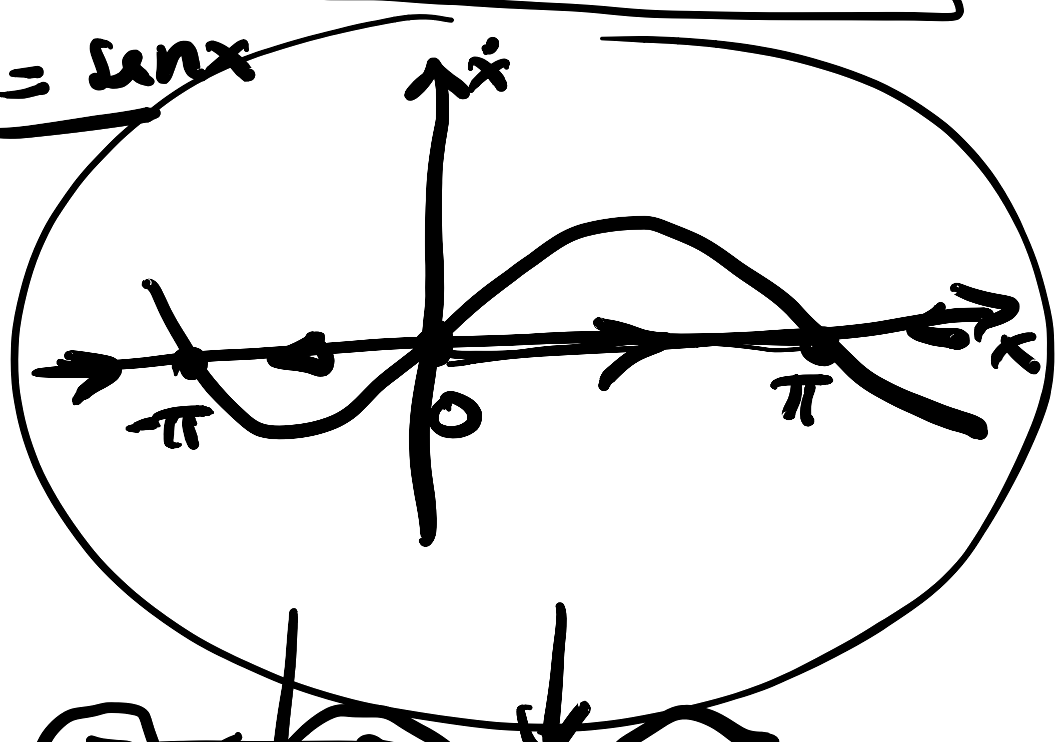
$$\boxed{t = \ln\left[\frac{(1+\cos x_0)\sin x}{(\sin x_0)(1+\cos x)}\right]}$$




# Geometrical Approach

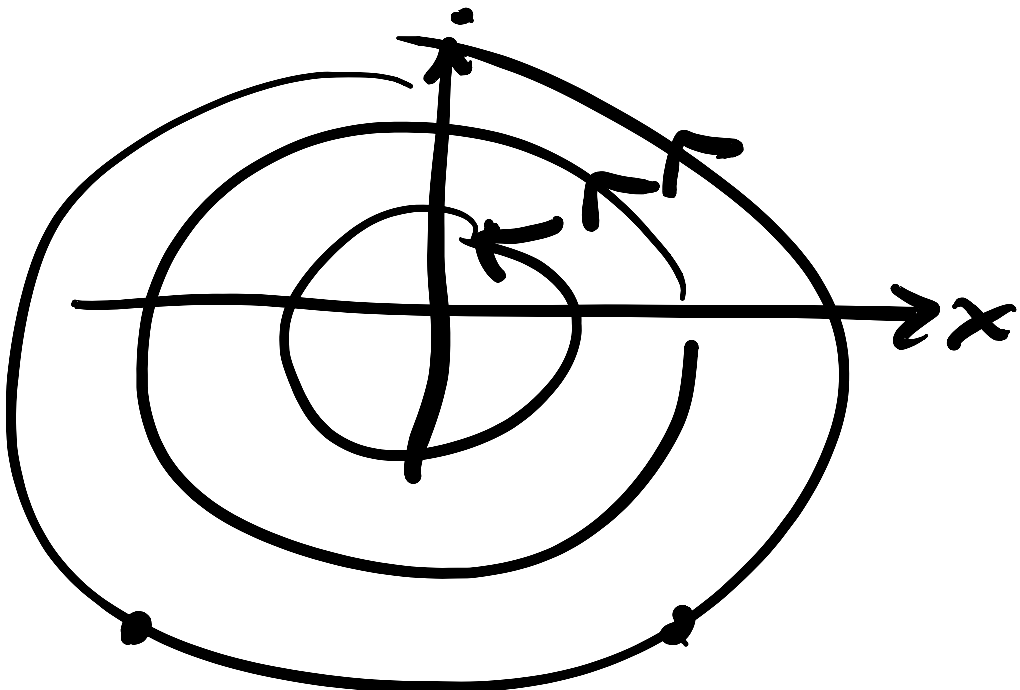
## Poincaré

$\dot{x} = \sin x$

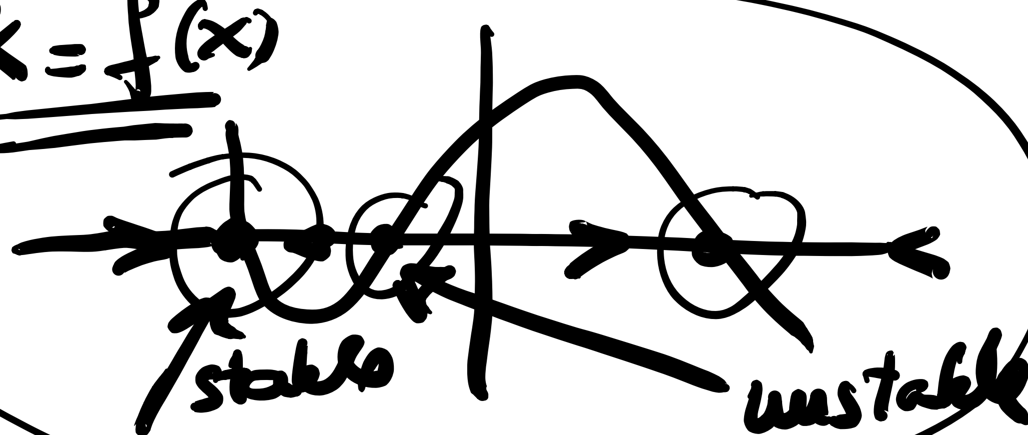


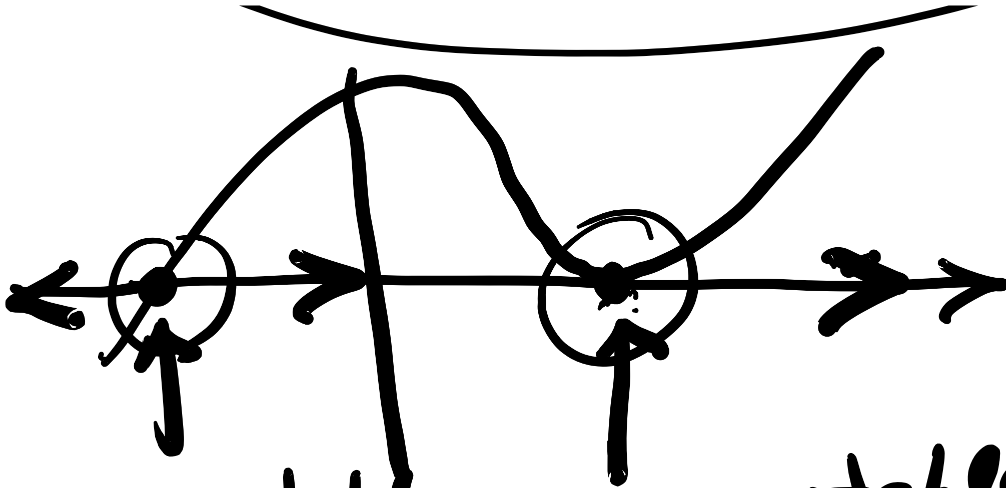


$m\ddot{x} + kx = 0$ $\quad x, \dot{x}$



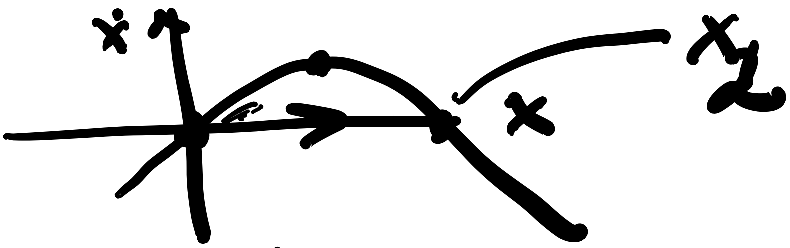

$\dot{x} = f(x)$





unstable

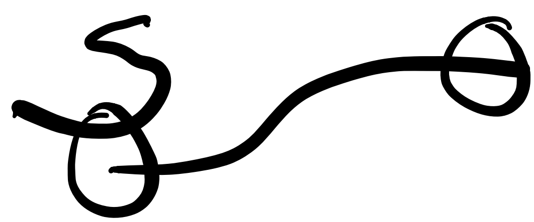
semi-stable
[unstable]



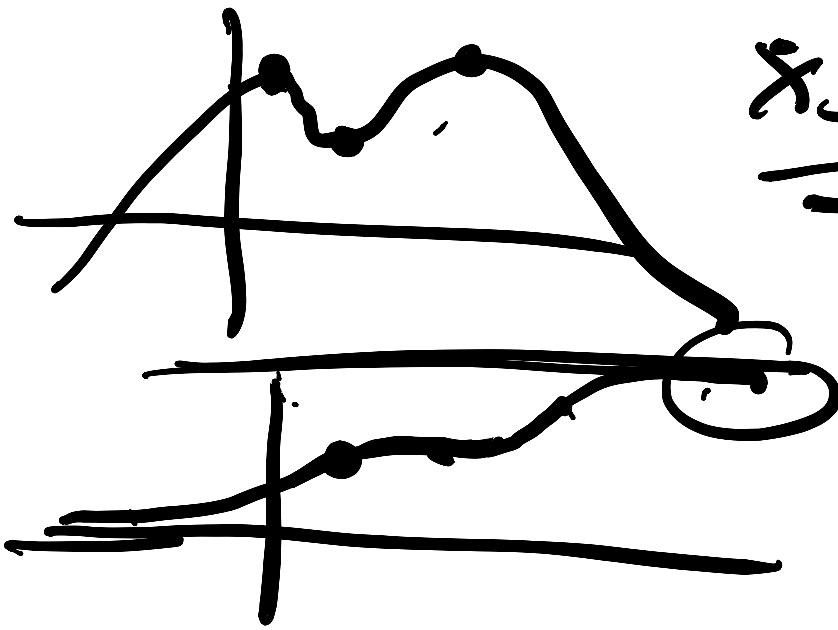


Inflection Point

Sigmoids



$\dot{x} = f(x)$



$\dot{x} = f(x)$ $\quad f(x_0) = 0$

$x = x_0 + \delta x$ $\quad \delta\dot{x} = f'(x_0)\,\delta x$

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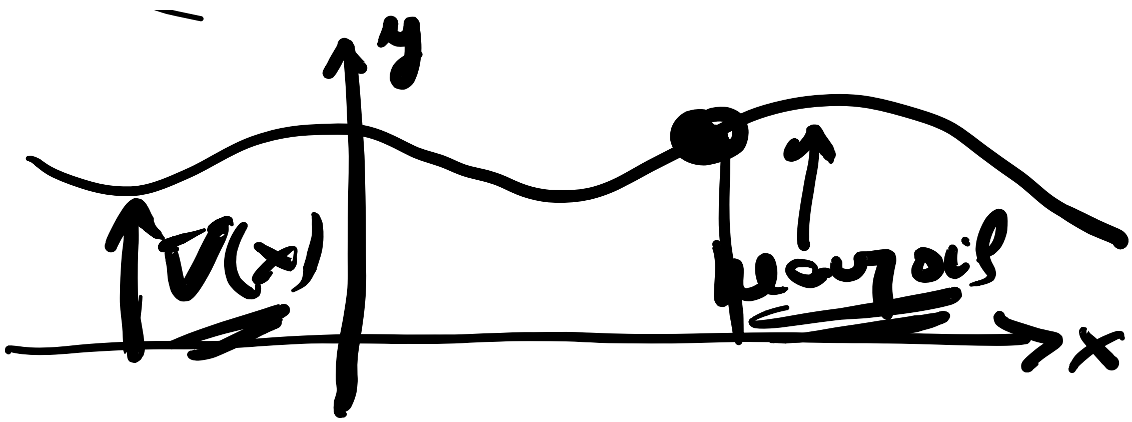
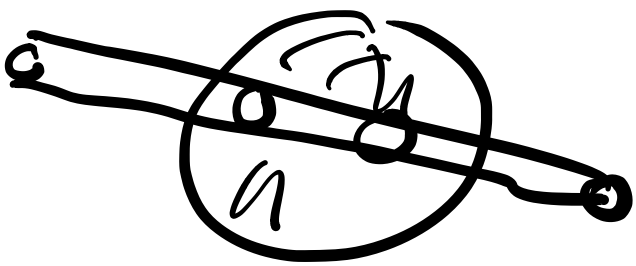
$\dot{y} = f(y, \lambda)$



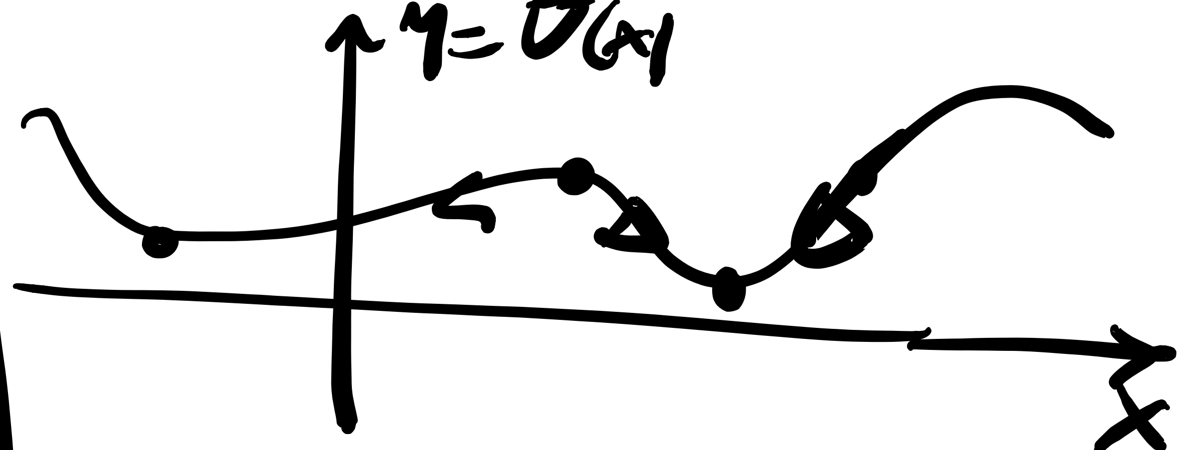



abacus

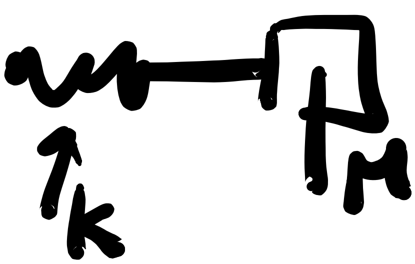
$$\cancel{m\ddot{x}} + \nu\dot{x} = -\frac{\partial V}{\partial x}$$

$y = V(x)$ shape of wire

$$\boxed{\ddot{x} = -V'(x)}$$




$$m\ddot{x} + b\dot{x} + kx = 0$$




$$\tilde{t} = T\,t$$

dim time

nondim

$$\boxed{T = b/k}$$

$$\cancel{\epsilon\ddot{x}} + \dot{x} + x = 0$$

$$\boxed{\epsilon = mk/b^2}$$

$$0 < \epsilon \ll 1$$