18.366 Random Walks and Diffusion — Spring 2003 – `http://math.mit.edu/18.366`

Instructor: Prof. Martin Z. Bazant, bazant@mit.edu. Office hours: M 2-3, Th 3-4 in 2-363B.
Teaching Assistant: Jaehyuk Choi, jaehyuk@mit.edu. Office hours: MW 11-12 in 2-331.
Time & Place: Tuesdays & Thursdays 11:00am–12:30pm, Room 2-147.
Prerequisite: 18.305 or permission of the instructor. A basic understanding of probability, partial differential equations, transforms, complex variables, asymptotic analysis, and computer programming would be helpful, but an ambitious student could take the class to learn some of these topics. *Interdisciplinary registration is encouraged.*

Problem Sets: Five, due on Thursdays Feb 13, 27, Mar 13, Apr 3, May 1. Solutions should be clearly explained. You are encouraged to work in groups and consult various references (but *not* solutions to 18.325 problems from 2001), although you must prepare each solution independently, in your own words.
Scribe Reports: Each student will write a detailed summary (ideally in LaTeXusing a template provided) of one or two lectures and/or solutions for selected homework problems. Each such report is due within one week, in a form that can be posted at the web site.
Midterm Exam: A take-home midterm exam will be handed out on Tu Apr 15 and turned in (at lecture) on Th Apr 17.
Final Project: There is no final exam, only a written final-project report, due at lecture on Th May 15. The topic must be selected and approved by Th Mar 20.
Grading: Grading will be based on the problem sets (40%), scribe reports (5%), midterm exam (25%), and final project (30%).

Required Books: Lecture notes from 18.325 (Spring 2001) available at MIT CopyTech (or at `http://math.mit.edu/~bazant/teach/18.325`; Sidney Redner, *A Guide to First Passage Processes* (Cambridge, 2001).
Recommended Books: Barry Hughes, *Random Walks and Random Environments*, Vol. 1 (Oxford, 1996); J. Crank, *Mathematics of Diffusion* (Oxford, 2nd ed., 1975); D. Ben-Avraham and S. Havlin, *Diffusion and Reactions in Fractals and Disordered Systems* (Cambridge, 2000); J.-P. Bouchaud and M. Potters, *Theory of Financial Risks* (Cambridge, 2000).

Description: Various mathematical aspects of (discrete) random walks and (continuum) diffusion will be developed in the context of real applications in physics, chemistry, and economics.

1. Simple Random Walks, Normal Diffusion, and the Central Limit Theorem (Structure functions, Edgeworth/Gram-Charlier expansions, uniformly valid saddle-point asymptotics, continuum limit, Kramers-Moyall expansion)

2. Return and First Passage (arcsine law, number of returns, first-passage time distributions, continuum limit for mean time)

3. Breakdown of the CLT and Anomalous Diffusion ('fat' power-law tails, strong CLT, Lévy flights, extreme events; nonidentical steps; correlated steps – persistence, self-avoidance)

4. Continuous-Time Random Walks

5. Random Walks with Drift (Einstein relation, advection-diffusion, Taylor dispersion)

6. Interacting Random Walks and Nonlinear Diffusion (short-range forces – granular drainage; long-range forces – electrochemical transport; reaction-diffusion, coalescence; fractal growth, DLA)