Nonlinear Dynamics: Chaos
Term Paper or Course Project [TPCP]

The aim of a term paper is for you to learn about a topic related to the course by reading some relevant material [papers or sections of book]. A secondary aim is for you to practice how to write mathematics or science in a way that is clear and logical. *The aim of a course project* is for you to implement and explore different methods for studying nonlinear and chaotic problems; either numerically or analytically; using methods (or variations of methods) seen in the course. The *distinction between a term paper and a course project is fuzzy,* and not very relevant.

I do NOT expect (though do not preclude) original research. Advice: Do NOT try to solve the hardest possible problem that you can imagine! I strongly encourage you to start your study from an established set of equations and start by reproducing well-known results. Once you have covered the fundamentals of the problem then you are free to explore the problem as much as you wish, and to consider variations on the original problem. It's easy to be overly ambitious in an attempt to impress, but *this is not the purpose of this.* What I want to see is that you understand, and can implement, methods from the course to a specific problem.† And, of course, show that you are able to write the results of your work in a way that is clear and logical.

† Important question arising in any course you take: *when do you know that you understand the material?*
A good answer is: when you can use it for problems <u>not</u> solved in the class lectures, notes, or book.

For the mechanics of the TPCP (deadlines, length, approval, etc) see the syllabus. *Extra stuff:*

Format: Typed and pdf. I strongly suggest that you use Latex, but other programs that allow for reasonable formatting of mathematical equations are perfectly fine.

What to include: Introduce the problem and possible methods for studying the problem numerically or analytically. Briefly explain how these methods work (if you wish to include lots of details then it is better to have an appendix). Discuss your results – carefully selected pictures go a long way! Then end with a Conclusions section and the References. **Important: cite all the sources,** and acknowledge authorship of any ideas or direct quotes — plagiarism is a serious offense!. Finally, attach a copy of any code used — this code does not count towards the page limits.

What I would like to see: *Well-written discussions of the problem and implementation; clear figures with readable axes and good captions.*

Possible projects

You are free to choose a project based on any material that is related to the content of this course. To reiterate: please do not be overly ambitious! I suggest doing a project that interests you as it helps with motivation. Here are some suggestions if you are lacking inspiration:

- A study of the Chaotic Waterwheel introduced by Strogatz in Chapter 9. Such a study would include some analysis of the fixed points and numerical investigation into the chaotic behavior. The analysis could be based on the analysis of the Lorenz equations.
- A study of the classical pilot-wave system explored in the recent paper *Bifurcations and chaos in a Lorenz-like pilot-wave system* by M. Durey, in "Chaos: An Interdisciplinary Journal of Nonlinear Science", Vol. 30, No. 103115, 2020 (October). The framework for this study could be similar to the aforementioned Chaotic Waterwheel. You can find more papers related to this topic in the "Focus issue: Hydrodynamic Quantum Analogs", September 2018, of this same journal. Or browse this journal for many more papers with interesting topics.

- A study of the Three-Body problem for planetary motion. The framework for this study could be similar to the Chaotic Waterwheel.
- Variations upon the *Romeo and Juliet* problems, including the role of nonlinear effects. Developing a system in which stable limit cycles exist might be particularly interesting.
- Explore the dynamics of the Spruce-Budworm problem. Strogatz book considers the case when the forest population is fixed, but one can also include the slow evolution of the forest (see exercise 8.1.10 on page 288 of Strogatz). The paper corresponding to this study is *Qualitative Analysis of Insect Outbreak Systems: The Spruce Budworm and Forest* by Ludwig, Jones and Holling (The Journal of Animal Ecology, Vol. 47, No. 1. (Feb., 1978), pp. 315-332), which is free to download on the internet.
- An analytical and numerical investigation into perturbations methods for analyzing limit cycles in nonlinear systems, similar to the analysis done in the lectures/notes for the van der Pol oscillator.

Topics by students in past years have included [partial list]

- The dynamics of the inverted driven pendulum.
- Nonlinear behavior of transverse vibration in axially loaded beams.
- A look into the nonlinear dynamics governing the Belousov Zhabotinsky reaction.
- An exploration of nonlinear dynamics in fuel cells.
- Exploring the nonlinear dynamics of ship motion.
- Applications of nonlinear dynamics and chaos to finance.
- Nonlinear dynamics of natural hazards: forest fires.
- Sunspot predictions for solar cycle 25 and non-linear dynamics.
- Double pendulum, Kapitza's pendulum, and double Kapitza's pendulum.
- Lyapunov exponents and breast cancer detection.
- An analysis of population dynamics within continuous viral particle production.
- Compartmental models for the spread of infectious diseases.
- The three-body problem and its associated nonlinearities.
- Numerical simulation and analysis of the three body problem.
- Analysis of nonlinear dynamics in electric systems.
- Non-linear dynamics of quantum dot lasers.
- Maps and plasma physics.
- Pseudorandomness and chaos.

I will be available for discussing possible projects at office hours, via e-mail, or individual Zoom meetings if necessary. Note that **you must have your project approved by November 8, and it is due the day before Thanksgiving.**

Acknowledgement

I would like to thank M. Durey, the 2020 18.353 lecturer, for the bulk of the contents of this description. I merely adapted it for the purposes of this year.