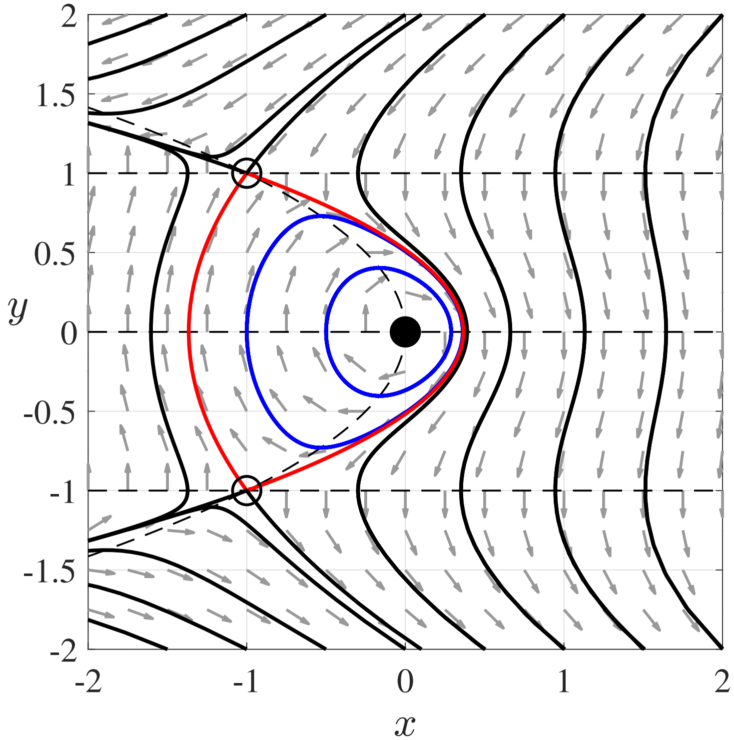

Phase portrait for the system

$$\dot{\boldsymbol{x}} = \boldsymbol{f(x,y) = y - y^3}, \quad \dot{\boldsymbol{y}} = \boldsymbol{g(x,y) = -x - y^2}.$$

This **system is reversible** due to invariance under the mapping $\boldsymbol{t \mapsto -t}$ and $\boldsymbol{y \mapsto -y}$, indicating that there is symmetry about $\boldsymbol{y = 0}$, but with the flow arrows pointing in the opposite direction.

The nullclines $\boldsymbol{y = 0}$, $\boldsymbol{y = \pm 1}$ (for $\dot{\boldsymbol{x}} = \boldsymbol{0}$) and $\boldsymbol{y = \pm\sqrt{-x}}$ (for $\dot{\boldsymbol{y}} = \boldsymbol{0}$) are denoted by dashed curves.

Linear stability analysis shows that there is a center at $\boldsymbol{(0,0)}$ and saddles at $\boldsymbol{(-1,\pm 1)}$. As the system is reversible, the origin is a *nonlinear center*, with closed trajectories highlighted in blue.

In this example, the manifolds of the saddle points connect, forming *heteroclinic trajectories* (highlighted in red).# All the closed orbits are confined within the region bounded by these heteroclinic trajectories.

This fact follows from the reversibility by an argument entirely similar to the one that guarantees that a linear center remains a center for a reversible system: Following forwards (resp.: backwards) in time the lower branch of the unstable (resp.: stable) manifold for the saddle at $(-1, 1)$ we can see that it reaches the x-axis. Reversibility then shows that it then continues symmetrically to reach the other saddle.