



Here are some suggestions on how to generate the phase portrait. The example considered here is

$$\dot{x} = f(x, y) = x + e^{-y}, \quad \dot{y} = g(x, y) = -y.$$

1. Plot the vector field (where the arrows are normalized to have equal magnitude) and the nullclines, $y = -\log(-x)$ (for $x < 0$) and $y = 0$ (dashed curves). By the shape of the vector field, we suspect a saddle point at the intersection of the nullclines (denote by the circle).
2. Add on some sample trajectories via simulation of the differential equations. I chose initial conditions on the edge of the domain where the vector field points towards the center of the plotted domain.
3. Compute the stable (black) and unstable (red) manifolds of the saddle. From the linear stability analysis, we have a local approximation for each manifold, which we can use as an initial condition for the unstable manifold. For the stable manifold, it is convenient to reverse time ($t \mapsto -t$) so that the system reads

$$\dot{x} = -f(x, y), \quad \dot{y} = -g(x, y).$$

Then evolve the time-reversed system from the linear approximation of the stable manifold.