Lecture 20 Thu 2021/11/18 ← Not on Web

Lecture 21 Tu 2021/11/23 ← Not on web

Continue with fractals

proof $[0,1]$ not enumerable (uncountable)

ω_0 = Nat #

reals $= 2^{\omega_0}$ = parts

hypothesis Continuum

reals $= \omega_1$

Not ω → Aleph! $\aleph_0 = N$ $N_1 = ??$

Cantor set describe { Structure at all scales; Self similar; non-integer dimension

Show

→ Measure zero

→ Uncountable ~ $[0,1]$

Convention $0.1 = 0.0222\ldots$

$C = \bigcap C_n$

How do we know not empty??

Define Self Similar dimension

length/size of copy = r

number of copies = $N(r)$

Note for square $N \sim r^{-2}$ | $r \downarrow 0$ | Make drawings!
cube $N \sim r^{-3}$

Define if $N \sim r^{-d}$ then d = dimension

$$d = -\log N / \log r \quad \lim r \downarrow 0$$

Cantor $r = 1/3$ $N = 2$ | $\left(\frac{1}{3}\right)^n = r$

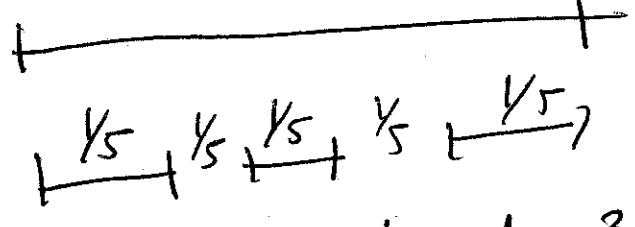
$r = 1/9$ $N = 4$ | $N = 2^n$

$$\therefore \boxed{d = +\log 2 / \log 3 \approx 0.63}$$

Koch curve Length $= \infty$ $\boxed{ds = \infty}$

$$d = \log 4 / \log 3 \sim 1.26$$

Even-5th Cantor



$N \sim 3^n$

$r \sim (1/5)^n$

$$d = \log 3 / \log 5$$

Topological cantor set

① Totally disconnected
all points separated from each other (no intervals)

② No isolated points
Points spaced apart but packed together

| Topological maps NOT Geometry

Often appear as strange attractors

Non self similar fractal

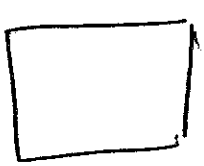
Cantor, but do not take middle third!

For example π = ternary notation

if nth digit 0 take left side
1 " middle
2 " right

Dimension?

Do also "gasket"



etc

use π with 9-adic !!

Box dimension

$N = N(\epsilon)$ = number of boxes covering

ϵ = size of box [say balls]

Then $N \sim \epsilon^{-d}$ d = dimension

$$d = -\frac{\ln N}{\log \epsilon} \quad \epsilon \downarrow 0$$

Cantor Set → get same answer as before.

Koch snowflake → " " "

Dimension $\boxed{\#1}$ → $N \sim 8^n$

$r \sim (1/3)^n$

$$-\frac{\ln N}{\log r} = \frac{\log 8}{\log 3}$$

Box dimension returns = 1 !!

not good !!

Box - in general hard to find minimal cover
- very expensive to compute
Conjures Q & R! [Hausdorff but even harder]

Tell "Why" of dimension

Pointwise Good for fractal that is limit of an orbit

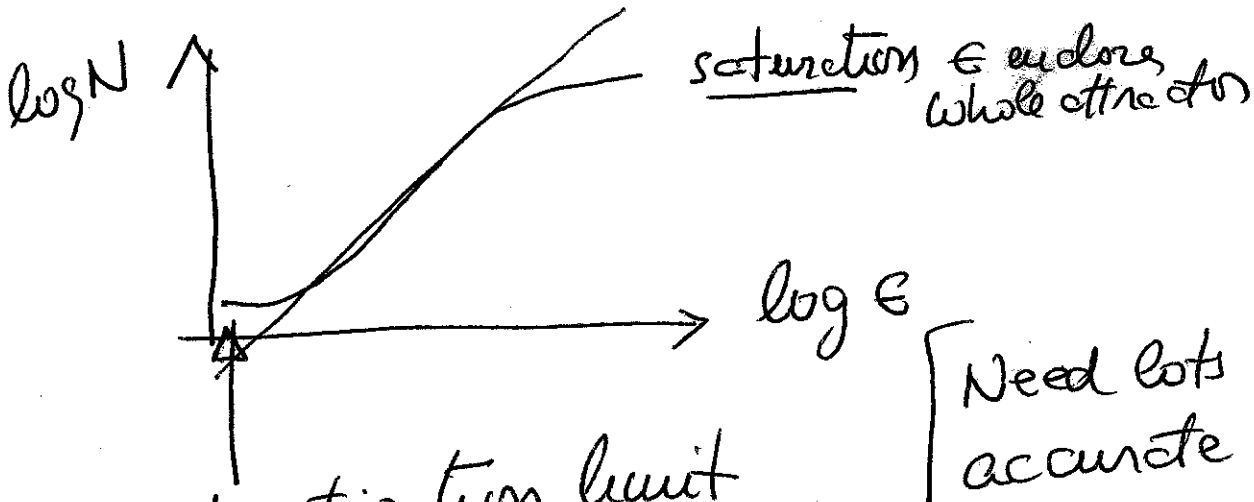
x_n or $x(t_n)$

Then $N(\epsilon)$ = number of point inside ball of radius ϵ centered at some x_n

$N \sim \epsilon^d$ — d = pointwise

Average over many points — correlation

$\log N \sim d \log \epsilon$



Need lots of accurate data to get a clear signal!

$$\Delta x << \epsilon << \text{diameter } A$$

$$d_{correlation} \leq d_{Box}$$ Theorem (but usually close)

Lorentz $d_{corr} = 2.05 \pm 0.01$

Logistic map $d_{corr} \approx 0.538 \pm 0.005$

↑ Topological cantor set $d_{box} \approx 0.538$

Now use for Rossler