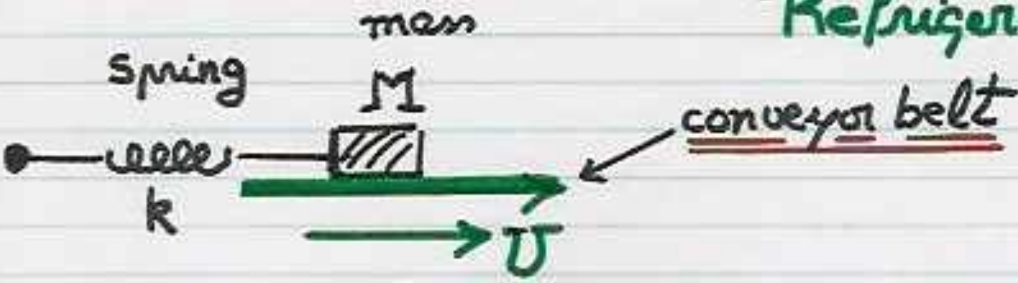
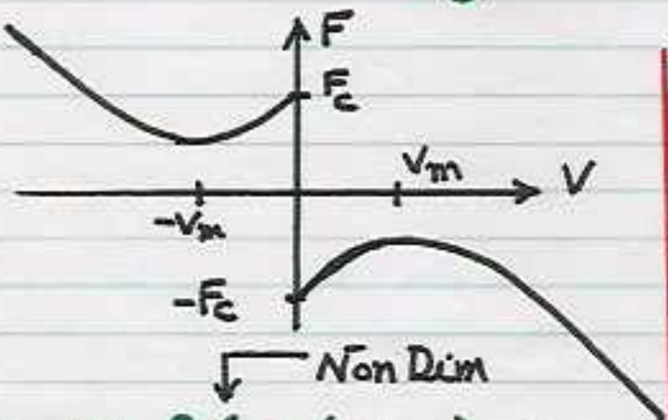
Relaxation oscillations due to friction

Refrigerator Pushing



Toy Model



$$F = F_c\, f(V/V_m)$$

Friction function as function of velocity. The existence of a static critical load F_c and a critical velocity V_m are important. $x=0$: spring relaxed

<u>Equation</u> $$M\ddot{x} = -kx + F(\dot{x} - U)$$

Equillibrium $x = x^*$, where $kx^* = F(-U)$

so, write $x = x^* + X$

Then $$M\ddot{X} = -kX + F(\dot{X} - U) - kx^*$$

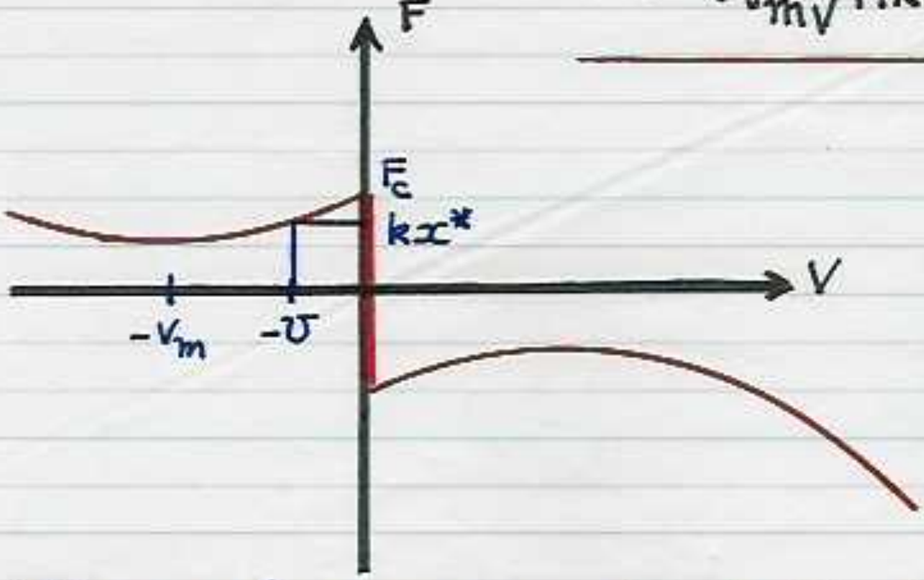
<u>Assume now</u>: $0 < U < V_m$, $U = O(V_m)$

and nondim. $x = (F_c/k)\,\tilde{x}$; $x^* = \frac{F_c}{k}\tilde{x}^*$;

$t = \underbrace{\sqrt{\frac{M}{k}}}_{\text{mass-spring response time.}}\,\tilde{t}$; $U = V_m \tilde{U}$

Then: $\ddot{x} = -x + f\left(\frac{F_c}{V_m\sqrt{Mk}}\dot{x} - U\right) - x^*$

drop ~ in nondim variables.

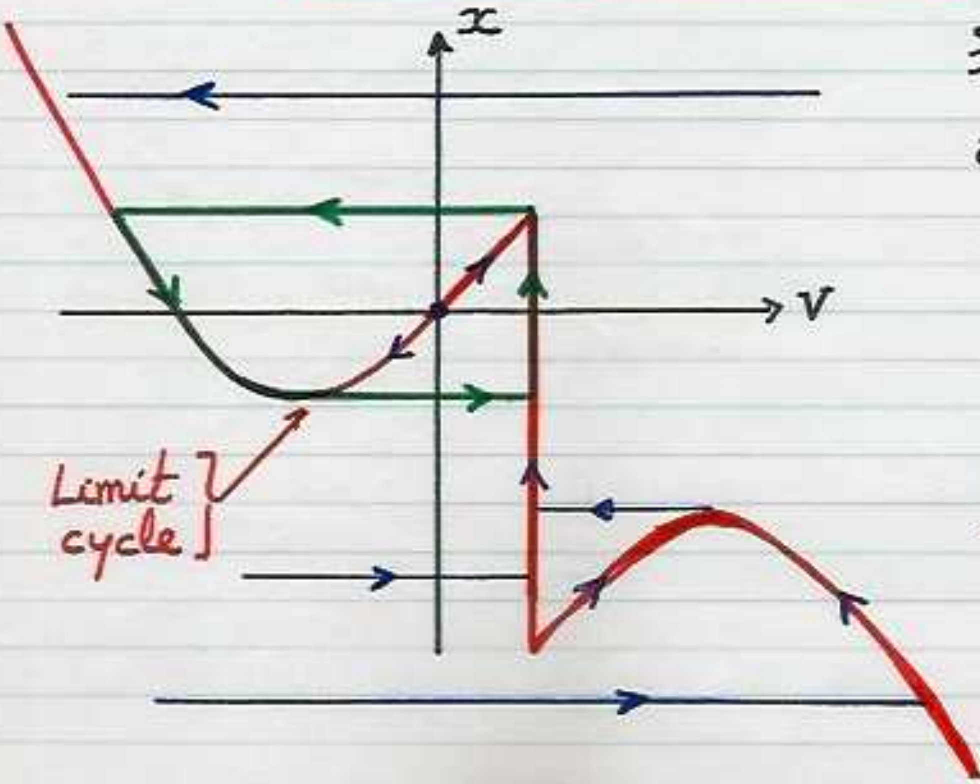


$$\boxed{\mu = \frac{F_c}{V_m\sqrt{Mk}}}$$

Non Dim parameter

In non-dim version : $F_c = 1$, $V_m = 1$, $F = f$
$k = 1$, $0 < U < 1$.

Let now $g(v) = f(v - U) - x^*$



$\ddot{x} + x = g(\mu\dot{x})$

or

$$\begin{cases} \dot{x} = \frac{1}{\mu} V \\ \dot{V} = \mu(g(V) - x) \end{cases}$$

For $\mu >> 1$

Thus, the conditions under which a relaxation limit cycle occur are:

$$0 < U < v_m \quad \left\{ \begin{array}{l} \text{Mean pushing} \\ \text{velocity in} \\ \text{critical range} \end{array} \right.$$

and $$F_c \gg v_m \sqrt{Mk}$$

This seems to indicate that the mass cannot be very large. How-ever $F_c \approx M\,a$

a typical acceleration

So get $\sqrt{M} \gg v_m \sqrt{k}/a$

F_c depends on weight!