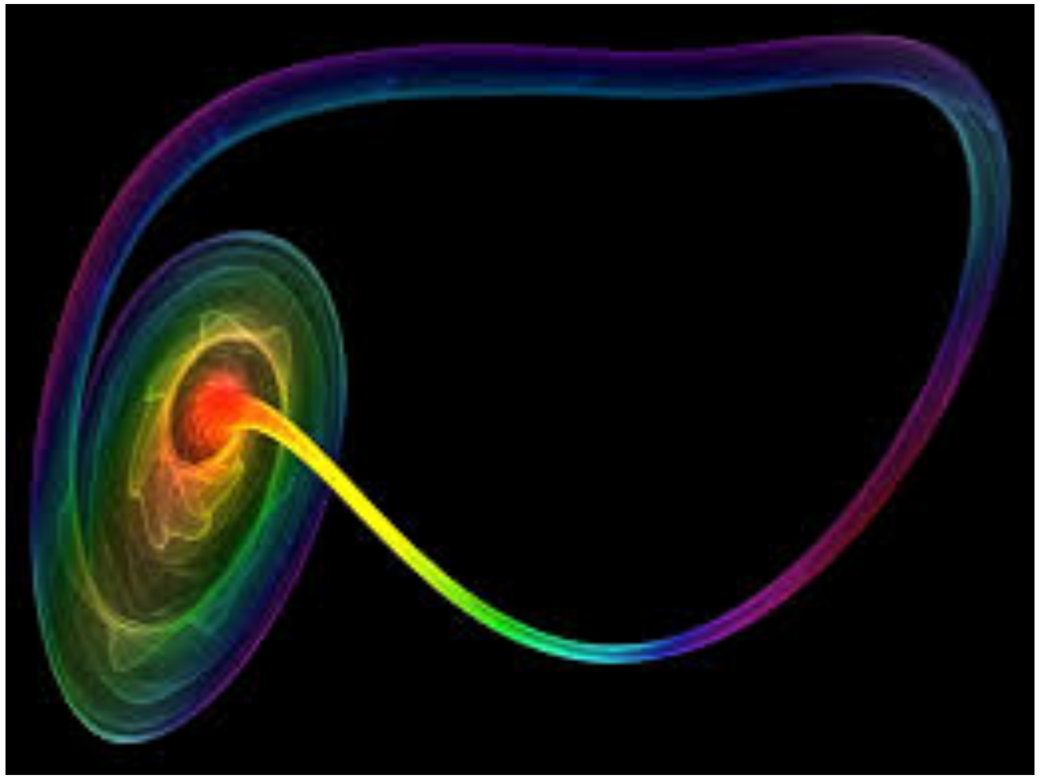
Nonlinear dynamics: Chaos
2.050/12.006/18.353

Introduction:
From the Fall 2020 term (by Matt Durey)
The basics of the class will not change much for the Fall 2021, so I decided to recycle this, removing things that are not relevant for this term.



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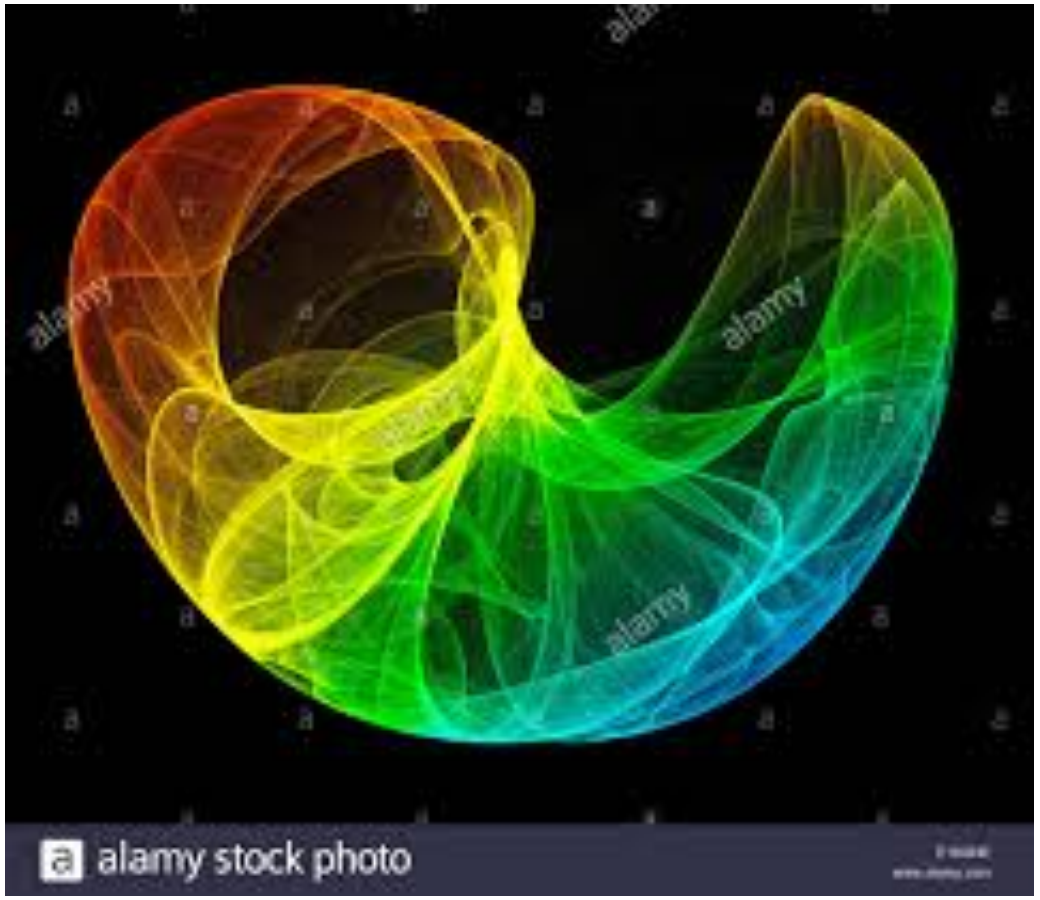


For details about the class, such as grading and "term paper"/"course project", please see the syllabus.

Linear equations

Since Newton and Leibniz independently developed the theory of calculus in the mid 17th century, a great deal has been understood about linear differential equations

E.g. $$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + b\frac{\mathrm{d}x}{\mathrm{d}t} + x(t) = F(t)$$

$$x(0) = x_0$$
$$\frac{\mathrm{d}x}{\mathrm{d}t}(0) = v_0$$

Methods of

- Find a homogeneous and particular solution
- Express as a matrix-vector system and use fundamental matrices

Linear ODE defintion: An ODE is linear if two solutions

$$x(t) \qquad y(t)$$

may be combined to give a third solution

$$z(t) = \alpha x(t) + \beta y(t)$$

for arbitrary constants $\alpha,\ \beta$

Nonlinear equations

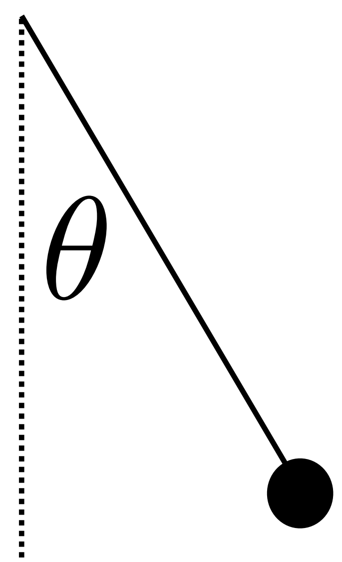
What happens when the drag depends quadratically on the speed?

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + b\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + x(t) = F(t)$$

Can we still analyze the dynamics of this system?

Pendulum

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{L}\sin\theta = 0$$



Only some nonlinear ODEs can be solved analytically, and often the answer is too messy to interpret! We instead seek simple graphical methods.

We aim to understand the solution of nonlinear ODEs
doing as little math as possible!

Chaotic dynamics

When the present determines the future, but the approximate present does not approximately determine the future. (E. Lorenz)

In other words, when the long-time dynamics of a system is highly sensitive to the initial conditions:
The Butterfly Effect

"Remember that hurricane a thousand miles away? That was me!"

All chaotic systems are described by nonlinear equations, but not all nonlinear equations yield chaotic dynamics