MIT Spring 2009 – Course Information
18.336 – Numerical Methods for Partial Differential Equations

Benjamin Seibold

1 General Information

Lecturer: Benjamin Seibold, 2-346, 324-2614, `seibold@math.mit.edu`
Office hours: Tuesdays, Thursdays, 12:30-1:30pm

Textbooks: The lecture is inspired by the following books, which are all recommended:

- Randall J. LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations — Steady State and Time Dependent Problems*, SIAM, 2007
- Randall J. LeVeque, *Finite Volume Methods for Hyperbolic Problems*, Cambridge University Press, 2002
- C.A.J. Fletcher, *Computational Techniques for Fluid Dynamics I*, Springer
- C.A.J. Fletcher, *Computational Techniques for Fluid Dynamics II*, Springer
- C.G. Canuto, M.Y. Hussaini, A. Quarteroni, T.A. Zang, *Spectral Methods, Evolution to Complex Geometries and Applications to Fluid Dynamics*, Springer, 2007

Course web page: `http://math.mit.edu/classes/18.336/`

Lectures: 2-136, Tuesdays, Thursdays, 11:00-12:30

Grading: Grading will be based on homework and course projects.

Problem sets: On average one problem set every two weeks, with exercises on theory and programming (50% of work load).

Course projects: To be worked on over the whole term, starting in second week. See Section 3 for more information (50% of work load).

Exams: There will be no exams.

2 Syllabus

This course addresses graduate students of all fields who are interested in numerical methods for partial differential equations, with focus on a rigorous mathematical basis. Many modern and efficient approaches are presented, after fundamentals of numerical approximation are established. Of particular focus are a qualitative understanding of the considered partial differential equation, fundamentals of finite difference, finite volume, finite element, and spectral methods, and important concepts such as stability, convergence, and error analysis.

Problems: advection equation, heat equation, wave equation, Airy equation, convection-diffusion problems, KdV equation, hyperbolic conservation laws, Poisson equation, Stokes problem, Navier-Stokes equations, interface problems.

Concepts: consistency, stability, convergence, Lax equivalence theorem, error analysis, Fourier approaches, staggered grids, shocks, front propagation, preconditioning, multigrid, Krylov spaces, saddle point problems.

Methods: finite differences, finite volumes, finite elements, ENO/WENO, spectral methods, projection approaches for incompressible flows, level set methods, particle methods, direct and iterative methods, multigrid.

3 Grading Policy

The grading will be based on homework (50%) and a course project (50%). The project consists of midterm report (20%), final report (60%), and presentation (20%).

Homework exercises There will be five problem sets with two weeks time to work on each. Exercises will be both theoretical and programming. The exercises will be involved, so you are advised to start immediately when the problem set is out. A typical feature of numerical analysis is that there is seldom one unique best method for a given problem, and the homework problems will reflect this aspect. Discussions with classmates and inside office hours are encouraged. However, each participant has to submit his/her own solution.

The obvious rules apply, i.e. copying of solutions is illegal, late submits have to be granted in advance, submitted program codes have to run without errors for grading, etc.

Course projects Over the course of the lecture time, each participant works on an extended problem related to the content of the course. You are allowed to choose a project which is related to your thesis work, however, the following restrictions apply:

- The project must focus on computational aspects related to the lecture topics.
- It is illegal to "reuse" a project from another course or thesis work. Your 18.336 project must cover specific questions and goals, which must not to be identical to the questions and goals of your thesis. For instance, you can investigate a specific computational aspect of your work, and investigate this aspect deeper than you would in your thesis work.

Of course, you can also choose a topic unrelated to your other work. For instance, you can consult the lecturer for interesting problems related to his research.

Any course project has to be agreed on by, and is under the supervision of the lecturer. The following deadlines apply:

Till second week (02/13): Submit project proposal (see below)

Before spring break (03/20): Submit midterm report on project

Last weeks of lecture time: Give short talk on project

Till beginning of last week of classes (05/11): Submit final project report

Note that **no** extensions can be given to these deadlines due to grading requirements.

3.1 Project Proposal

Email your project proposal to `seibold@math.mit.edu` with the following information:

Project title

Project background Does it relate to your work in another field (e.g. your thesis)? If yes, briefly outline the questions and goals of your work in the other field.

Questions and Goals Briefly describe the questions you wish to investigate in your project. What are your expectations?

Plan Which language do you plan to program in? Do you intend to use special software? Does your project work relate to the work of other people at MIT?