MIT Dept. of Mathematics
Benjamin Seibold

18.336 spring 2009
Problem Set 2

Out Thu 02/26/09 Due Thu 03/12/09

Problem 4
Consider the PDE

$$\begin{cases} -\varepsilon u_{xx} + u_x = 1 & \text{in }]-1,1[\\ u(-1) = 0 \ , \ u(1) = 0 \end{cases} \tag{1}$$

with $\varepsilon = 10^{-4}$.

1. Implement a simple finite difference method (using equidistant grids), and run an error analysis. Explain the observed behavior.
2. Write a spectral code using Chebyshev points. You can use the function `cheb.m` by Nick Trefethen, as linked on the course web site.
3. Write a finite difference code that uses less than 250 grid points and approximates the solution with accuracy $\|u_{\text{approx}} - u_{\text{true}}\|_\infty < 10^{-4}$.

Problem 5
Consider the Poisson equation on a periodic domain

$$\begin{cases} -u_{xx} = f & \text{in } [0, 2\pi] \\ u^{(k)}(0) = u^{(k)}(2\pi) & \forall k \geq 0 \end{cases}$$

with f periodic on $[0, 2\pi]$.

1. Show that solutions only exist if f satisfies a condition. Show further that if a solution exists, there is a one parameter family of solutions.
2. Write a second order accurate finite difference code for this problem that yields the solution u with zero mean.
3. Write a spectral code for this problem that yields the solution u with zero mean. Use the Matlab programs `p4.m` and `p5.m` as inspiration. Both programs are by Nick Trefethen, as linked on the course web site.
4. Run an error analysis for both your codes for the following right hand sides:
   (a) $f(x) = \sin(x) + 5\sin(18x) + 5\sin(20x)$
   (b) $f(x) = \begin{cases} 1 & x \in [\frac{\pi}{2}, \frac{3\pi}{2}[\\ -1 & \text{otherwise} \end{cases}$