18.306 Advanced Partial Differential Equations with Applications.

Tue. and Thu. 9:30–11:00 in 2–139

Description: The focus of the course are the concepts and techniques for solving the pde (partial differential equations) that permeate various scientific disciplines. Applications include problems from fluid dynamics, electrical and mechanical engineering, materials science, quantum mechanics, etc.

Prerequisites: Basic theory of one complex variable and a good working knowledge of ode (ordinary differential equations).

Textbook: No required textbook. However, the first part (roughly first 2/3) of the course will be based on chapters 1–5 of the book by S. Salsa in the textbook list below.

Instructor: R. R. Rosales, 2-337, x3-2784, rrr@math.mit.edu. Off. hrs TBA in web page.

TA: TBA.

Exams: None. Neither midterm, nor final. See **GRADING** below.

Problem sets: About 7 ± 1 problems sets (one every 1-2 weeks). **Do them all:** *you need them to learn the material* — see **GRADING** below. Complete answers will be posted after their due date.

Term paper: One is needed. See **GRADING** below.

WEB page: http://www-math.mit.edu/18.306/index.html

MatLab: I strongly urge you to become proficient in MatLab. See the course web page for more information. You can install MatLab in your own computer if you purchase the **Student version of MatLab.**

News Updates: ***Check the News Updates link in the web page frequently!***
Information concerning office hours, TA, etc. will be posted there.

GRADING: Each problem set will contain a buried mini-quiz within it (1-3 problems). Only the quiz will be graded, but you need to do all the problems, since the quiz problems will not be identified. The course grade will be based on the cumulative quiz-grade, and a binary-grade term paper.
Term paper grading: If you hand in an acceptable term paper, then the grade is quiz-based, as explained above. Else the grade is F. The term paper can be handed any time before the last day of lectures.

TERM PAPER. Can be on any topic relevant to the course materiel (instructor pre-approval required). It does not have to be original research, but it must be original work [e.g.: review the literature in some topic, and summarize the results in your own words, giving proper credit to the sources]. *The explanations must be clear, and accessible by someone with the level of an average student in the class!* You can use materiel from your own research, but "recycling" (e.g.: handing a piece of your thesis) is not allowed. You must process it to follow the guidelines here. Further requirements:

- Must be typed (font size 12-14) and submitted electronically in pdf format.
- Length should not exceed about 15 pages, using standard page formatting. You can use more if you have many figures, but use judgement here!

Failure to follow all these guidelines may result in a paper being classified as "not acceptable".

Recommended books and textbooks.

1. Whitham, G. B. *Linear and nonlinear waves.* John Wiley, 1999.
2. Salsa, S. *Partial differential equations in action.* Springer, 2008 (revised reprint 2009).
3. Debnath, L. *Nonlinear partial differential equations for scientists and engineers.* Birkhauser, 1997.
4. Guenther, R. B., and Lee, J. W. *Partial Differential Equations of Mathematical Physics and Integral Equations.* Dover.
5. Levine, H. *Partial differential equations.* Am. Math. Soc.: International Press, 1997.
6. Kevorkian, J. *Partial differential equations: analytical solution techniques.* Springer, 2000, 2nd ed.
7. Carrier, G. F., and C. E. Pearson. *Partial differential equations: theory and technique.* Academic Press, 1988, 2nd ed.
8. Hinch, E. J. *Perturbation methods.* Cambridge U. Press, 1991.
9. Barenblatt, G. I. *Scaling, self-similarity, and intermediate asymptotics.* Cambridge U. Press, 1997.
10. Drazin, P. G., and R. S. Johnson. *Solitons: An Introduction.* Cambridge U. Press, 1989.

Books with more emphasis on rigorous mathematical theory.

11. Evans, L. C. *Partial Differential Equations.* Am. Math. Soc., 1998.
12. DiBenedetto, E. *Partial Differential Equations.* Birkhauser, 1994.
13. Garabedian, P. R. *Partial Differential Equations.* Am. Math. Soc., 1998.

OUTLINE of the Course: A rough idea follows. Some things may be covered in more detail than this implies, or the reverse. In fact: it is unlikely that there will be enough time to cover everything. This is just to give you an idea of the "flavor".

- **Introduction.** Terminology; boundary and initial value problems; well- and ill-posed problems.
- **First-order pde.** Complete solutions; characteristics; conservation laws; systems of pde; introduction to weak solutions: shocks and jump conditions; entropy condition; examples: traffic flow, gas dynamics, etc.
- **Linear pde.** Review and classification; Laplace, wave and diffusion equations; Klein-Gordon equation; more on characteristics; standard methods: separation of variables, integral transforms, Green's functions; potential scattering; special topics in conformal mapping; dispersion and diffusion; dimensional analysis and self-similarity; regular and singular perturbation theory; asymptotics for complete solutions; geometrical optics and WKB. eikonal equation; high-frequency expansions; caustics;
- **More on nonlinear pde.** Equations that convert into linear pde; some exactly solvable cases; Burgers' equation; dimensional analysis and similarity; traveling waves; nonlinear diffusion and dispersion; the KdV, nonlinear Schrdinger and Sine-Gordon equations; reaction-diffusion equations; Fisher's equation; singular perturbations: boundary layers, homogenization, weakly nonlinear geometrical optics, etc.; Solitons; Backlund transformations; Painlevé conjecture.
- **Variational Methods.** First and second variation; Euler-Lagrange equation; constraints.
- **Free-boundary value problems.** Formulation; perturbation theory; more on water waves; method of extended gradient; materials surface evolution; some open problems.