18.305 FALL 2005 — PROBLEM SET #2
M.I.T. Department of Mathematics
POSTED: Friday, 10/07/05 **DUE:** Friday, 10/14/05
Turn-in place: Math. Undergrad. Office, Rm. 2-108.

1. Find the leading term for each of the integrals below by using the Laplace method for $\lambda \gg 1$:

   (a) (10 points)
$$\int_{-2}^{1} dx\ e^{-\lambda x^3}(1+x^2).$$

   (b) (10 points)
$$\int_{1}^{\infty} dx\ e^{-\lambda \cosh x}\sqrt{x^2-1}.$$

   (c) (10 points)
$$\int_{0}^{1} dx\ e^{\lambda x^2(1-x)}.$$

2. Find the leading term for each of the integrals below by using the Laplace method for $\lambda \gg 1$:

   (a) (10 points)
$$\int_{-\pi/2}^{\pi/2} dx\ e^{-\lambda \cos x}.$$

   (b) (10 points)
$$\int_{0}^{\infty} dx\ e^{-\lambda(x+x^5)}.$$

3. (20 points) Find the *entire* asymptotic series for the integrals 2(a), (b) above.

4. Find the leading term for each of the integrals below by using the stationary-phase method for $\lambda \gg 1$:

   (a) (10 points)
$$\int_{-a}^{a} dt\ e^{i\lambda t^5}, \quad a > 0.$$

   (b) (10 points)
$$\int_{1}^{\infty} dt\ e^{i\lambda t^4}\sqrt{1+t^2}.$$

   (c) (10 points)
$$\int_{0}^{\pi} dt\ e^{i\lambda \sin t}.$$

CONTINUED ON REVERSE:

5. **SPECIAL PROBLEM:** (a) (10 points) Find the *entire* asymptotic series expansion for the integral

$$I = \int_{-\infty}^{\infty} dt\; e^{i\lambda \sinh^4 t}, \qquad \lambda \gg 1.$$

(b) (10 points) Find an upper bound for the remainder (error) of the expansion in part (a) when N terms are summed where $N \gg 1$. Explain what N you would choose for maximum accuracy in evaluating I. **Hint:** You may wish to use Stirling's formula, $\Gamma(z) \sim \sqrt{2\pi}\, e^{-z}\, z^{z-1/2}$ as $|z| \to +\infty$ and $|\mathrm{Arg}\, z| < \pi$.