18.300 Principles of Continuum Applied Mathematics
Term Paper or Course Project [TPCP]

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The project's aim is for you to implement and explore various aspects of the course materiel: modeling of physical set-ups where the techniques introduced in the course are useful, and/or where the physical phenomena studied occur. It is also an opportunity to see how various methods (analytical and numerical) are used in actual research.

A secondary (though no less important) objective is to learn and practice bibliographical searches. If you do one of the projects suggested below, do not limit yourself to the references given. **Explore and find more!** However: look for reliable references; e.g.: papers in refereed journals. Youtube videos and other random web stuff can be fun, but generally you have no idea of how much the author knows about the subject, the methods used to arrive at conclusions, etc. Journals have experts review the papers before publication (not fool-proof, but it helps).

I DO NOT expect (though do not preclude) original research. DO NOT try to solve/investigate the hardest problem you can imagine! I strongly encourage you to start from established knowledge, reproducing and explaining well-known results. Once you have covered the fundamentals of the problem then you are free to explore as much as you wish and to consider variations on the original question. It is easy to be overly ambitious in an attempt to impress, but I would rather see that you understand and can apply the course methods to a specific topic.

The **TPCP must obey the following guidelines:**

- Instructor pre-approval is required. This **must be done by by Fri April 15.**
- The **due date is Fri April 29,** when you must upload it to the canvas webpage.
- Typed (font size 11-12) and submitted in **pdf format**. Use LateX to produce the pdf file.
- Length should not exceed 10 pages (less is better, I suggest about 5), using standard page formatting. You can use more if you have many figures, but use judgement here!
- *Explanations must be clear, accessible to someone with the level of an average student in the class.*
- *Be certain to give proper credit to your sources.*

What to include: Introduce the subject/topic and possible methods for studying the problem numerically or analytically. Briefly explain how these methods work (if you wish to include lots of details do it in an appendix). Discuss results – pictures go a long way! Conclusion. References (**please cite all sources** - plagiarism is a serious offense!). Print-out of code (or the main parts), if any – this does not count towards the aforementioned 10 pages length. **What I like:** Well-written discussion of the topic; clear figures with readable axes.

Possible topics

You are free to choose a project based on any material related to the course's content. To reiterate: do not be overly ambitious! I suggest doing a project that interests you as it helps with motivation. Suggestions follow (note that **some of the topics are too big, and you can select only a fraction for your TPCP**).

- **Shock Wave Lithotripsy.** Check here for a medical description of the condition. [1]
  Below a paper with a fluid dynamics investigation of the condition:

  Progress in Modeling and Simulation of Shock Wave Lithotripsy (SWL), M. Tanguay and T. Colonius. Fifth International Symposium on Cavitation (CAV2003) Osaka, Japan, November 1-4, 2003.
  https://colonius.caltech.edu/pdfs/TanguayColonius2003.pdf

  See if you can find more. **Related to this is:**#

  The focusing of weak shock waves, B. Sturtevant and V. A. Kulkarny. Journal of Fluid Mechanics, Vol. 73 #4, pp. 651–671 (1976). # The relationship is that Lithotripsy involves focusing of strong acoustical waves on the kidney stones, and the focusing generates shear waves (which damage tissue).

- **Shocks and Syringomyelia.** Check here for a medical description of the condition. [2]
  See what you can find on this. Here is a lead:

  Hypothesis on the pathophysiology of syringomyelia based on simulation of cerebrospinal fluid dynamics, H. S. Chang and H. Nakagawa. Journal of Neurology, Neurosurgery and Psychiatry Vol. 74, pp. 344–347 (2003).

[1] https://my.clevelandclinic.org/health/treatments/16582-shockwave-lithotripsy

[2] https://www.mayoclinic.org/diseases-conditions/syringomyelia/symptoms-causes/syc-20354771

- **Traffic Flow: Second Order Models, Phantom Traffic Jams, Jamitons.**

  Payne-Whitham model (first second order model).

  Models of freeway traffic and control, H. J. Payne. Proc. Simulation Council, Vol 1 pp. 51–61, (1971).

  Linear and nonlinear waves, G. B. Whitham. John Wiley and Sons, New York, (1974).

  FREEFLO: A macroscopic simulation model of freeway traffic, H. J. Payne. Transp. Res. Rec., Vol 722: pp. 68–77, (1979).

  Requiem for second-order fluid approximations of traffic flow, C. F. Daganzo. Transp. Res. B, Vol. 29, pp. 277–286 (1995).

  Aw-Rascle-Zhang model (next second order model).

  Resurrection of "second order" models of traffic flow, A. Aw and M. Rascle. SIAM J. Appl. Math., Vol. 60, pp. 916-938 (2000).

  Extension and amplification of the Aw-Rascle model, J. M. Greenberg. SIAM J. Appl. Math., Vol. 62, pp. 729-745 (2001).

  A non-equilibrium traffic model devoid of gas-like behavior, H. M. Zhang. Transp. Res. B, Vol. 36, pp. 275-290 (2002).

  Congestion redux, J. M. Greenberg. SIAM J. Appl. Math., Vol. 64, pp. 1175–1185 (2004).

  Phantom Traffic Jams, Jamitons.

  Self-sustained nonlinear waves in traffic flow, M. R. Flynn, A. R. Kasimov, J.-C. Nave, R. R. Rosales, and B. Seibold. Phys. Rev. E 79, 056113 (May 2009), or `https://arxiv.org/abs/0810.2820`.

  Constructing set-valued fundamental diagrams from jamiton solutions in second order traffic models, B. Seibold, M. R. Flynn, A. R. Kasimov, and R. R. Rosales. Networks and heterogeneous media, American Institute of Mathematical Sciences, Vol. 8, # 3 (September 2013), or `https://arxiv.org/abs/1204.5510`.

- **Simple models for Detonation waves.**

  Detonation in miniature, W. Fickett. Am. J. Physics Vol. 47, # 12, pp. 1050-1059, (Dec. 1979).

  Model for Shock Wave Chaos, A. R. Kasimov, L. M. Faria, and R. R. Rosales. Phys. Rev. Lett. 110, 104104 (March 2013), or `https://arxiv.org/abs/1202.2989`

  Stability of a square wave detonation in a model system, W. Fickett. Physica D, Vol 16 #3, pp. 358-370 (1985).

  If you are interested in analysis and proofs, check: *Stability of detonations in the ZND limit,* K. Zumbrun. `https://arxiv.org/abs/0906.2821` and related work by the same author.

- **Circular hydraulic jumps. Termination Shock of the Solar Wind.**

  A stationary circular hydraulic jump, the limits of its existence and its gasdynamic analogue, A. R. Kasimov. J. Fluid Mech., Vol. 601, pp. 189–198 (2008). Check references in this paper to prior work; the topic has a long history.

  Search for analogies to hydraulic jumps for the following Astrophysical topics: V-waves, bow shocks, termination shock, etc. Below is a random example:

  The Termination Shock of the Solar Wind, M. A. Lee. Space Science Reviews, Vol. 78, # 1-2, pp. 109–116 (1996).

- **Roll waves.**

  Mathematical solution of the problem of roll-waves in inclined open channels, R. F. Dressler. Communications on Pure and Applied Mathematics Vol. 2, # 2-3, pp. 149–194 (1949).

  Dynamics of roll waves, N. J. Balmforth and S. Mandre. J. Fluid Mech., Vol. 514, pp. 1–33 (2004).

  Nonlinear Stability of Viscous Roll Waves, M. A. Johnson, K. Zumbrun, and P. Noble. SIAM J. Math. Anal. Vol. 43, No. 2, pp. 577–611 (2011).

- **Continuum modeling of networks.**

  Continuum modeling and control of large nonuniform networks, Y. Zhang, E. K. P. Chong, J. Hannig and D. Estep. 2011 49th Annual Allerton Conference on Communication, Control, and Computing (Allerton), 2011, pp. 1670–1677.

Approximating Extremely Large Networks via Continuum Limits, Y. Zhang, E. K. P. Chong, J. Hannig, D. Estep. Access IEEE, Vol. 1, pp. 577–595, (2013).

Model of the Information Shock Waves in Social Network Based on the Special Continuum Neural Network, A. Bomba, N. Kunanets, V. Pasichnyk and Y. Turbal. 2018 IEEE First International Conference on System Analysis and Intelligent Computing (SAIC), 2018, pp. 1–5.

- **Kinematic waves with a neither convex, nor concave, flux.**

  A kinematic wave model for rivers with flood plains and other irregular geometries, P. M. Jacovkis and E. G. Tabak. Math. Computing and Modelling, Vol. 24, # 11, pp. 1–21 (1996).

- **A few more.** I have not checked these in any detail, but wikipedia includes reliable references.

  Detonation Engine. Wikipedia: `https://en.wikipedia.org/wiki/Rotating_detonation_engine`

  Prince Rupert's Drops. Wikipedia: `https://en.wikipedia.org/wiki/Prince_Rupert%27s_drop`

  Water Hammers. Wikipedia: `https://en.wikipedia.org/wiki/Water_hammer`

  Note: if you choose one of these topics, read some papers. Do not do it relying on wikipedia. In particular, for water hammers, I want to know about the science and the engineering (not plumbing).

The topics below are less directly related to the course material (no shocks; though see paper #5 in Reaction-Diffusion), and involve pde types we have not looked much at, but should still be accessible to you.

- **Solitons, Inverse Scattering Transform.**

  Integrals of nonlinear equations of evolution and solitary waves. P. Lax. Communications on Pure and Applied Mathematics, Vol. 21, #5, pp. 467–490, (1968)

  Method for Solving the Korteweg-deVries Equation, C. S. Gardner, J. M. Greene, M. D. Kruskal, and R. M. Miura. Phys. Rev. Lett., Vol. 19, 1095 (November 1967).

  The inverse scattering transform-Fourier analysis for nonlinear problems, M. J. Ablowitz, D. J. Kaup, A. C. Newell, and H. Segur. Stud. Appl. Math., Vol. 53, pp. 249–315 (1974).

  On the complete integrability of a nonlinear Schrödinger equation, V. E. Zakharov and S. V. Manakov. Journal of Theoretical and Mathematical Physics, Vol. 19, #3, pp. 551–559. (1974).

- **Reaction-Diffusion systems; Excitable media; Morphogenesis; Spiral Waves.**

  The chemical basis of morphogenesis, A.M. Turing. Philosophical Transactions of the Royal Society (part B), Vol. 237, pp. 37–72 (1952). This one is a classic. Start here and look for more on the subject.

  Varieties of spiral wave behavior: An experimentalist's approach to the theory of excitable media, A. T. Winfree. Chaos, Vol. 1, #3 pp. 303–334 (1991).

  Spiral Waves of Chemical Activity, A. T. Winfree. Science, Vol. 175, pp. 634–636 # 4022 (1972).

  Rotating Chemical Reactions, A. T. Winfree. Scientific American, Vol. 230, # 6, pp. 82–95 (1974).

  Slowly Varying Waves and Shock Structures in Reaction-Diffusion Equations, L. N. Howard and N. Kopell. Stud. Appl. Math., Vol. 56, # 2, pp. 95–145 (1977).

  Rotating spiral wave solutions to reaction diffusion equations, D.S Cohen, J.C Neu, and R. R. Rosales. SIAM J. Appl. Math., Vol. 35, pp. 536–547 (1978).

  Propagating Fronts in Reactive Media, Paul C. Fife. Pages 267–285 in North-Holland Mathematics Studies, Non-linear Problems: Present and Future, Vol. 61, January 1982. Ed.: A. Bishop, D. Campbell, and B. Nicolaenko.

- **Tides and Kelvin waves.**

  Tidal oscillations in gulfs and rectangular basins, G. I. Taylor. Proceedings of the London Mathematical Society, Ser. 2, Vol. 20, pp. 148–181 (1922).
  Taylor was inspired by the English channel, but similar ideas have been applied to other geographical regions. See if you find some.

I will be available for discussing possible projects at office hours, via e-mail, or individual meetings if necessary. Please e-mail me a short paragraph describing the planned outline of your project, so I can approve it by the deadline **Friday April 15.** I have to read it, and either approve it or suggest modifications, so do not wait till the last minute.