Plancherel's Theorem [redacted]

Define the Fourier Transform by

$$\hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx \quad (1)$$

Then

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(k) e^{ikx} dk \quad (2)$$

Theorem If f is square integrable,
then $\hat{f}$ is square integrable (T1)
and $\int |f|^2 dx = \int |\hat{f}|^2 dk$ (T2)

Intuitive proof: In the space of square integrable functions, consider the standard scalar product $\langle f, g \rangle = \int \bar{f}(x) g(x) dx$. Then (1) defines the F.T. as a linear operator $\mathcal{L} f = \hat{f}$ and (2) says that $\mathcal{L}^{+} = \mathcal{L}^{-1}$. That is: $\mathcal{L}$ is unitary, which gives (T2).