18.300 Principles of Continuum Applied Mathematics.

Tue and Thu 9:30-11:00 in room 2-131.

Quotes and anecdotes you should keep in mind.
Chinese proverb: *He who asks a question is a fool for five minutes; he who does not remains a fool forever.*
Chinese proverb: *The master leads you to the door, the rest is up to you.*
Anecdote involving Ernest Rutherford (1871-1937), a famous New Zealand physicist. The story is told that one student in his laboratory (in England) was extraordinarily hard-working. Having taken note of this, Rutherford asked him one evening, "Do you work in the mornings too?" The student (expecting commendation) proudly replied: "Yes!" Rutherford then looked puzzled, and asked in amazement: "But when do you think?"

Covid-19: This course will be taught in person. However, a few lectures will be virtual [if this is better for the material covered, or because the lecturer is sick]. All the **lectures will be recorded,** and made available to the students through the canvas site: The in person lectures via panopto and the virtual ones via zoom.

Webpages: Two web pages: one public in the Math. site, and another in canvas [canvas: upload problem set answers, grading, announcements, piazza, recordings, etc.]. The **main page** is the public one, to which canvas is linked at various places. Please **check this webpage regularly.** Main web page: https://math.mit.edu/classes/18.300/index.html

Textbook: There is no single textbook. The material in the course is, unfortunately, spread out over several books. **Class notes** for some topics will be made available in the course webpage. See the references at the syllabus' end for a book list (all on library reserve).

Pre-requisites: Calculus II (GIR) and (18.03 or 18.032). Some physical intuition, and elementary physical notions will be needed.

MatLab: I **urge you to become proficient in MatLab.** MatLab course scripts will be used in some lectures, and you should use them to reinforce the course materiel. MatLab will also be needed for some problem sets. See the course web page for more MatLab information.

Team work: I encourage you to form study groups. We will talk about this during the first lecture. PSet Partners https://psetpartners.mit.edu can be helpful to find study partners.
Please **read the POLICIES entry in the web page.**

Instructor: R. Rosales, rrr@math.mit.edu, room 2-337.
Off. Hours: TBA, **check the Office Hours entry in Canvas.**
Easiest way to contact me is by email. If we need to meet, we can schedule a meeting this way. **Do not just drop by my office (or call my office phone), I am rarely there.** *For technical questions, use piazza, so everyone has access to the answer.*

TA/grader: TBA.

Exams: None. Neither midterm, nor final. See **GRADING** below.

Problem sets: 8 (± 1) problem sets (one every ≈ 1.5 weeks). See **GRADING** below.
Do them all: *you need them to learn the material.* **You will need a computer, and MatLab.** Complete answers will be posted after their due date.
The problem set answers have to be uploaded to the canvas site in **pdf** format. They **must be typed, I will provide a LaTeX template that you can use for this purpose.**

TPCP: You will need either a **Term Paper** or a **Course project**. See **GRADING** below.

GRADING: Each problem set will contain a buried mini-quiz within it (1-3 problems). Only the quiz will be graded, but you need to do all the problems, since the quiz problems will not be identified. The course grade will be based on the cumulative quiz-grade, and a binary-grade for the TPCP (acceptable/not acceptable).
TPCP grading: If you hand in an acceptable TPCP the grade is quiz-based, as above. Else the grade is F.

TPCP. On any topic relevant to the course materiel. It does not have to be original research, but it must be original work. Examples: (i) *Review the literature in a topic related to the course material, and summarize the results in your own words.* (ii) *A short report discussing an application of course techniques to a physical/biological/etc system that you find interesting.* The topic or model does not need to be complex. This can be an original topic, or a review of an existing application. Possible topics include specific applications to: aerodynamics and fluid flow, astrophysics, biological system models, chemical reactions, classical mechanics, ecosystem models, epidemiology, sports, and transportation. **I will post a pdf file with some suggestions in the web page.**

The **TPCP must obey the following guidelines:**

- Instructor pre-approval is required. This **must be done by by Fri April 15.**
- The **due date is Fri April 29,** when you must upload it to the canvas webpage.
- Must be typed (font size 11-12) and submitted in **pdf format**.
- Length should not exceed 10 pages (less is better, I suggest about 5), using standard page formatting. You can use more if you have many figures, but use judgement here!
- *The explanations must be clear, and accessible by someone with the level of an average student in the class.*
- *Be certain to give proper credit to your sources.*

Failure to follow all these guidelines may result in a paper being classified as "not acceptable".

The distinction between "term paper" and "course project" is fuzzy. *This is intentional.* I want to give you freedom to be creative. Propose something somewhat "open ended", different from solving a bunch of homework problems, and which require you to plan and write a coherent exposition of some interesting topic, and I will probably approve it. Note also that it should be **short,** do not make this into an expedition to the South Pole.

OUTLINE of the Course: Some things may be covered in more detail than this implies, or the reverse.

PART I. Some basic topics in Nonlinear Waves.

- Shock waves and hydraulic jumps. Description and various physical set ups where they occur: traffic flow, shallow water. What is a wave.
- Fundamental concepts in continuous applied mathematics. Continuum limit. Conservation laws, quasi-equilibrium. Kinematic waves.
- Traffic flow (TF). Continuum hypothesis. Conservation and derivation of the mathematical model. Integral and differential forms. Other examples of systems where conservation is used to derive the model equations (in nonlinear elasticity, fluids, etc.)
- Linearization of equations of TF and solution. Meaning and interpretation. Solution of the fully nonlinear TF problem. Method of characteristics, graphical interpretation of the solution, wave breaking. Weak discontinuities, shock waves and rarefaction fans. Envelope of characteristics. Irreversibility in the model.
- Quasilinear First Order PDE's.
- Shock structure, diffusivity. Burger's equation. The Cole-Hopf transformation. The heat equation: derivation, solution, and application to the Burger's equation. Inviscid limit and Laplace's method.
- Dimensional analysis. Nonlinear diffusion and fronts.

PART II. Numerical solutions, series, and transforms.

- Numerical solution of wave equations. Finite differences. Good and bad numerical schemes: consistency, stability, von Neumann analysis. Associated equation. Short wave stability analysis.
- Computers and numerical issues. MatLab.
- Fourier series and von Neumann stability. Discrete and Fast Fourier transforms. Spectral methods.
- Transforms and series: Fourier, Laplace.

- Lattices. Fermi-Pasta-Ulam problem.

A few of these topics will be covered *if time permits*: • Sonic booms. Mach cone. • Caustics. • Shallow water waves. Equations. Linearization and solution. Radiation conditions. Characteristics and shocks. • Random walks, brownian motion, diffusion. • Water waves. Derivation of the equations and linearization. Dispersion and group speed. Weak nonlinearity and solitary waves. Perturbation expansions. • Linear and nonlinear oscillations, relaxation. Phase plane methods and multiple scales. Applications to celestial mechanics and mechanical vibrations. • Dynamical systems examples from mathematical biology and population dynamics.

Since there are no exams, the main things to keep in mind are: (i) a problem set every 1.5 weeks (approx.), with at least a week for each, from posting till due. (ii) the TPCP.

References (on library reserve):

- **R. Haberman**. *Mathematical Models, Mechanical Vibrations, Population Dynamics and Traffic Flow.* SIAM. ISBN 9780898714081. *Covers many course topics.* SIAM Classics in Applied Mathematics #21
- **R. Haberman.** *Applied Partial Differential Equations with Fourier Series and Boundary Value Problems.* Pearson Education. ISBN 9780134995434.
- **J. D. Logan**. *An Introduction to Nonlinear Partial Differential Equations.* Wiley. ISBN 9780470225950. *Covers the p.d.e. theory in this course – part I, at an easy level.* Volume 93 of Pure and Applied Mathematics: A Wiley Series of Texts, Monographs and Tracts.
- **G. B. Whitham.** *Linear and Nonlinear Waves.* Wiley. ISBN 9780471359425. *An extremely good book to have.*
- **C. C. Lin and L. Segal**. *Mathematics Applied to Deterministic Problems in the Natural Sciences.* SIAM. ISBN 9780898712292. *Another extremely good book to have.* SIAM Classics in Applied Mathematics #1.
- **F. Y. M. Wan**. *Mathematical Models and their Analysis.* Harper and Row. Covers some of the topics. SIAM Classics in Applied Mathematics #79.
- **R. D. Richtmyer and K. W. Morton.** *Difference Methods for Initial-Value Problems (2nd ed.).* Krieger Publishing Company. ISBN 9780894647635.
- **A. C. Fowler.** *Mathematical Models in the Applied Sciences.* Cambridge U. Press. ISBN 9780521467032.
- **J. J. Stoker.** *Nonlinear Vibrations in Mechanical and Electrical Systems.* Wiley.
- **S. Strogatz.** *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering.* CRC Press LLC. ISBN 9780738204536.

Other useful books:

P. G. Drazin. *Nonlinear Systems.* Cambridge U. P.

D. W. Jordan and P. Smith. *Nonlinear Ordinary Differential Equations.* Oxford U. P.

S.W. McCuskey. *Introduction to Celestial Mechanics.* Addison Wesley.

Coddington & Levinson. *Theory of Ordinary Differential Equations.* McGraw-Hill or Krieger.

A. J. Lichtenberg and M. A. Lieberman. *Regular and stochastic motion.*, Springer.

R. S. MacKay and J. D. Meiss. *Hamiltonian Dynamical Systems; a reprint selection.* CRC Press.

The End