

**18.158 Fall 2015. Topics in Differential Equations:
Free Boundary Problems**

Time and Place: 2:30-4pm, Tue, Th; E17-139.

Lecturer: David Jerison; E17-308.

The main goal of the course is to prove regularity of energy-minimizing free boundaries in dimensions 2, 3, and 4. These are results from 1984, 2002, and 2014, respectively. Regularity in dimensions 5 and 6 is an open problem; there are singular solutions in dimension 7 and higher.

We will start by discussing parts of the book by Caffarelli and Salsa “A Geometric Approach to Free Boundary Problems” (AMS Graduate Studies in Mathematics vol 68, 2005) which establishes partial regularity in all dimensions and covers what was known through 2000.

We will also explain the broad and deep analogy between the theory of level sets of solutions to semilinear elliptic partial differential equations, free boundary theory, and minimal surface theory. Regularity results for energy-minimizing free boundaries are inspired by the analogous theory of Jim Simons concerning regularity of area-minimizing surfaces.

The analogy with minimal surfaces leads to many other questions. For example, in the spirit of the theory of Colding and Minicozzi for minimal surfaces, one can try to classify singular limits of free boundaries that are not necessarily energy-minimizing. If time permits, we will reverse the flow of ideas and talk about open problems in the theory of minimal surfaces for which techniques from free boundary theory may be relevant. In particular, we will discuss isoperimetric surfaces, that is, least area surfaces dividing bodies into two parts of specified volume.

One example of a free boundary problem is to find the shape of insulating material with a fixed volume surrounding an oven or refrigerator that is optimal in the sense that it yields the least temperature flux. Other physically motivated free boundary problems are to find the outline of a flame or plasma, the shape of a jet from a water hose, or the wake of a boat.