Homework Assignment 4

Due Friday, October 24.

Exercise 1

Let $f_k \in C^{k-2}(\mathbb{R})$ be the function, $r^k \mathrm{Log\ r, r = |x|}$. Show that for $2N = k + n - 2$

$$\Delta^N f_k = \frac{C}{r^{n-2}},$$

C a nonzero constant. (Hint: Friedlander-Joshi, page 61, display 3). Conclude (loc. cit. display 4) that for $n > 2$,

$$\Delta^{N+1} f_k = C\delta_0.$$

Exercise 2

Part 1. Let $\mathcal{U}$ be an open subset of $\mathbb{R}^n$ and u an element of $\mathcal{D}'(\mathcal{U})$. Show that if u has the property $\mu(\varphi) \geq 0$ for real valued functions, $\varphi \in C_c^\infty(\mathbb{R}^n)$ with $\varphi \geq 0$, then μ is a distribution of order 0, i.e., a continuous map

$$C_c^0(\mathcal{U}) \to \mathbb{C}.$$

Hint: For K a compact subset of $\mathcal{U}$, let $f \in C_c^\infty(\mathcal{U})$ be a non-negative function which is identically one on a neighborhood of K and show that for $\varphi \in C_c^\infty(K)$

$$|\mu(\varphi)| \leq \mu(f) \sup(|\phi(x)|, x \in K)$$

Part 2. Conclude by the Riesz representation theorem (see Rudin's "Real and Complex Analysis") that there exists a measure, ν, on the Borel subsets of $\mathcal{U}$ such that for $\varphi \in C_c^\infty(\mathcal{U})$,

$$\mu(\varphi) = \int_{\mathcal{U}} \varphi \, dy$$

Exercise 3

Part 1. Reading Assignment: §3.2 in Friedlander-Joshi.

Part 2. Friedlander-Joshi, exercise 4.6.

Exercise 4

Friedlander-Joshi, exercise 5.6.

Exercise 5

Friedlander-Joshi, exercise 6.1.