18.152: Fall 2010
Homework 9: Last one!

Available	Tuesday, November 30	Due	Tuesday, December 7

Turn in the homework at the beginning of class on Tuesday, November 30. No late homework is accepted unless previously arranged with the instructor.
This week homework will cover old material and new material from (in order) pages 274–282, 249–257and 156–168.

1. Consider the problem

$$\begin{cases} \rho u_{tt} - \tau u_{xx} = \gamma u_{xxt} & 0 < x < L,\ t > 0 \\ u(x,0) = f(x), \quad u_t(x,0) = g(x) & 0 \le x \le L, \\ u_x(0,t) = u(L,t) = 0 & t \ge 0, \end{cases} \tag{0.1}$$

with ρ, τ, γ positive constants.

(a) Let

$$E(t) = \frac{1}{2}\int_0^L [\rho u_t^2 + \tau u_x^2]\, dx.$$

Assuming that u is regular enough, prove that $E'(t) \le 0$ and give an interpretation of this result.

(b) Use part (a) to prove uniqueness for solutions to problem (0.1).

(c) Using the method of separation of variables, prove that $u(x,t) \to 0$ for $t \to \infty$.

2. Problem 5.8 in textbook.

3. Problem 5.9 in textbook.

4. Problem 5.16 in textbook.

5. Problem 5.17 in textbook.

6. Problem 4.1 in textbook.

7. Problem 4.2 in textbook.