18.152: Fall 2010
Homework 5

Available	Tuesday, October 26	Due	Tuesday, november 2

Turn in the homework at the beginning of class on Tuesday, November 2. No late homework is accepted unless previously arranged with the instructor.
This week homework will cover old material and new material from 113-118 and 124-128.

1. Problem 3.6 in textbook

2. Problem 3.8 in textbook.

3. Problem 3.11 in textbook.

4. (a) Let g be a function in[1] $L^1(\mathbb{R}) \cap C(\mathbb{R})$ and it is bounded. Prove that there is a unique solution, bounded and continuous in the half plane $S = \{(x, y) \ / \ y \geq 0\}$, for the problem

$$\begin{cases} \Delta u = 0 & x \in \mathbb{R},\ 0 < y \\ u(x,0) = g(x) & x \in \mathbb{R} \\ u(x,y) \ \text{ bounded in } S. & \end{cases}$$

(b) using the partial Fourier transform given by

$$\hat{u}(\xi, y) = \int_{\mathbb{R}} e^{-i\xi x} u(x,y)\, dx$$

write a formula for the solution u.

5. Let u_i, $i \in \mathbb{N}$, be nonnegative harmonic functions on a domain Ω of $\mathbb{R}^2$. Using Harnack's inequality prove that if the series $\sum_{i=0}^{+\infty} u_i$ converges at a point x_0 of Ω, then it converges uniformly in any compact set $K \subset \Omega$. Deduce also that the sum of the series is a nonnegative and harmonic function.

[1] We say that a function g belongs to $L^1(\mathbb{R})$ if $\int_{-\infty}^{\infty} |g(x)|\, dx < \infty$.