18.152: Fall 2010
Homework 4

Available	Tuesday, October 12	Due	Tuesday, October 19

Turn in the homework at the beginning of class on Tuesday, October 19. No late homework is accepted unless previously arranged with the instructor.
This week homework will cover old material and new material from page 90 to page 96 and from 102-112.

1. Problem 2.23 in textbook

2. Answer the following questions with a full explanation:

(a) Let u be solution to the equation $u_t - u_{xx} = -1$ in $0 < x < 1$ and $t > 0$ such that

$$u(x,0) = 0 \quad \text{and} \quad u(0,t) = u(1,t) = \sin \pi t.$$

Is it possible that there exists a point $x_0,\ 0 < x_0 < 1$ such that $u(x_0, 1) = 1$?

(b) Does the Cauchy problem

$$\begin{cases} u_t(x,t) + u_{xx}(x,t) = 0 & -1 < x < 1,\ 0 < t < T \\ u(x,0) = |x| & -1 < x < 1 \\ u_x(0,t) = u(1,t) = 0 & 0 < t < T \end{cases}$$

have a solution? (*Hint*: look carefully at the equation!)

(c) Check that the function $u(x,y) = \partial_x \Gamma_1(x,t)$ solves the problem

$$\begin{cases} u_t(x,t) - u_{xx}(x,t) = 0 & x \in \mathbb{R},\ 0 < t \\ u(x,0) = 0 & x \in \mathbb{R} \end{cases}$$

and that $u(x,t) \to 0$ if $t \to 0$, for each fixed x. Is there a contradiction with the uniqueness theorem for the global Cauchy problem?

(d) Let $u(x,t)$ be the continuous solution of the Robin's problem

$$\begin{cases} u_t(x,t) + u_{xx}(x,t) = 0 & & 0 < x < 1,\ 0 < t < T \\ u(x,0) = \sin \pi x & & 0 \le x < 1 \\ -u_x(0,t) = u_x(1,t) = -hu, & h > 0 & 0 < t < T. \end{cases}$$

Show that u cannot have a negative minimum. What is the maximum for u?

3. Problem 3.1 in textbook.

4. Problem 3.2 in textbook.

5. Problem 3.4 in textbook.