18.152: Fall 2010
Homework 3

Available	Tuesday, October 5	Due	Tuesday, October 12

Turn in the homework at the beginning of class on Tuesday, October 12. No late homework is accepted unless previously arranged with the instructor.
This week homework will cover old material and new material from page 58 to page 76.

1. Let's consider again Problem 2.11 in textbook. Let $X(t)$ be the position of the particle at time t and T_L the first time when the article reaches L. We have that T_L is a random variable defined by

$$T_L = \inf_s \{X(s) = L\}.$$

(a) Show that in the limit the particle reaches L in finite time with probability 1, that is the probability of event $\{T_L < \infty\}$ is equal 1.
Hint: Show that

$$\text{Prob } \{T_L > t\} \to 0 \quad \text{if } t \to \infty.$$

You can show this easily if you relate this probability to the probability of the particle being in a certain interval.

(b) Compute the probability that the particle reaches L before time t, that is compute

$$F(L, t) := \text{ Prob } \{T_L \leq t\}.$$

(c) Determine the probability that the particle passes for the first time through $x = L$ in the interval of time $(t, t + dt)$.

2. Consider the problem:

$$\begin{cases} u_t(x,t) - u_{xx}(x,t) = 0 & x > 0, \ t > 0 \\ u(x,0) = 0 & x \geq 0 \\ u_x(0,t) = g(t) & t > 0, \end{cases}$$

(a) Using Fourier (cosine) transform solve the problem above where g is continuous and bounded in $L^2(0, \infty)$. Prove that this is the only solution.

(b) Prove that, without the condition that g is in $L^2(0, \infty)$, the problem above doesn't have a unique solution by using the two functions $w_1(x,t) = e^x \sin(2t + x)$ and $w_2(x,t) = -e^{-x} \sin(2t - x)$.

3. Problem 2.14 in textbook.

4. Problem 2.15 in textbook.

5. Problem 2.16 in textbook.