

18.152: Fall 2010

Homework 2

Available	Tuesday, September 28	Due	Tuesday, October 4
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Turn in the homework at the beginning of class on Tuesday, October 4. No late homework is accepted unless previously arranged with the instructor.

This week homework will cover the material from page 31 to page 57.

1. (Asymptotic behavior) Let u be a continuous solution on $S = [0, 1] \times [0, \infty)$ of the problem

$$\begin{cases} u_t(x, t) - u_{xx}(x, t) = 0 & 0 < x < 1, t > 0 \\ u(x, 0) = x(1 - x) & 0 \leq x \leq 1 \\ u(0, t) = u(1, t) & t > 0. \end{cases}$$

First show that $u \geq 0$. Then find the range of $\alpha > 0$ and $\beta > 0$ such that

$$u(x, t) \leq w(x, t) = \alpha x(1 - x)e^{-\beta t}.$$

Deduce from here that $u(x, t) \rightarrow 0$ uniformly in $[0, 1]$ as $t \rightarrow +\infty$.

Hint: Use the maximum principle twice, once with u and once with $w - u$.

2. Problem 2.8 page 98 of textbook.

Hint: Use chain rule when you take derivatives.

3. Problem 2.9 page 98 of textbook.

4. Problem 2.11 page 98 of textbook.

Hint: Everything is as in the textbook for $[0, L)$. The new ingredient is at $x = L$. Here write what $p(L - h, t + \tau)$ should be using the fact that in the previous time step it could only have come from space step $L - 2h$. Then take the limit.....hopefully I didn't confuse you further :-)!