18.152: Fall 2010
Homework 1

Available	Tuesday, September 21	Due	Tuesday, September 28

Turn in the homework at the beginning of class on Tuesday, September 28. No late homework is accepted unless previously arranged with the instructor.
This week homework will cover the material from Lectures 1-5

1. Prove that the function
$$f(x) = \frac{\sin x \log(x^2+1)}{|x|^{2-\epsilon}}$$
is a function in $L^2(\mathbb{R})$, (here $\mathbb{R}$ is the set of real numbers).
Hint: You have to recall the definition and prove the convergence of the integral as an improper integral. Be careful, there are two locations where you have to pay a lot of attention: at $x = 0$ and at infinity. Do not try to compute the integral!

2. Consider the full ellipse $\Omega = \{(x, y) \quad / \quad ax^2 + by^2 \leq r^2\}$, where $a, b > 0$ are fixed.

- Show that Ω is a bounded domain with smooth boundary.
- Set up the Gauss divergence formula with Ω as above and $\vec{F}(x, y) = x\vec{i} + 2y\vec{j}$.

Hint: Look up any multivariable calculus book.

3. Problem 2.2 page 97

4. Problem 2.3 page 97

5. Problem 2.4 page 97

6. Problem 2.6 page 97

7. Consider the heat equation on $(0, 1)x(0, \infty)$ with Dirichlet boundary conditions and initial data $u(x, 0) = 4x(1 - x)$ Show that $u(x, t) = u(1 - x, t)$ for all $t \geq 0, 0 \leq x \leq 1$.

8. [Robin boundary conditions and Energy Method] Consider the heat equation on$(0, L)\times(0, \infty)$with Robin boundary conditions, $u_x(0, t) - a_0 u(0, t) = 0, \quad u_x(L, t) + a_1 u(L, t) = 0$, for $a_0,\ a_1 > 0$. Use the energy method to show that the endpoints contribute to the decrease of $\int_0^L u(x, t)^2 dx$, i.e. the time derivative of this quantity is less than or equal to the time derivative one obtains in the case of Dirichlet boundary conditions.