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We first prove the hint. Let $g = \sum_{i=1}^n a_i 1_{E_i}$ be a simple function, wlog $\{E_i\}$ disjoint. Then

$$\int g d\nu = \int \sum_{i=1}^n a_i 1_{E_i} d\nu = \sum_{i=1}^n \int a_i 1_{E_i} d\nu = \sum_{i=1}^n a_i \nu(E_i)$$

$$= \sum_{i=1}^n a_i \int_{E_i} f d\nu = \sum_{i=1}^n \int f a_i 1_{E_i} d\nu = \int f \sum_{i=1}^n a_i 1_{E_i} d\nu = \int g f d\nu.$$

Now let g be a nonnegative mble function. Then $\exists$ a sequence $\{\phi_n\}$ of ^{nonneg., mble} simple functions s.t. $\phi_n \uparrow g$ (by pg. 44 of Stroock). As f is nonnegative and mble, $f\phi_n \uparrow fg$, and thus by

the MCT,

$$\int g d\nu = \int \lim_{n \rightarrow \infty} \phi_n d\nu \stackrel{\text{MCT}}{=} \lim_{n \rightarrow \infty} \int \phi_n d\nu$$

$$= \lim_{n \rightarrow \infty} \int \phi_n f d\nu$$

$$\stackrel{\text{MCT}}{=} \int \lim_{n \rightarrow \infty} \phi_n f d\nu$$

$$= \int f \lim_{n \rightarrow \infty} \phi_n d\nu = \int g f d\nu$$