

5.4 Since $\{f_n\}$ are mble, $f_n \rightarrow f$ μ -a.e., and

20/20

$|f_n| = f_n \leq f, f \in L^1(\mu)$ μ -a.e., by the DCT,

$$\lim_{n \rightarrow \infty} \int f_n d\mu = \int \lim_{n \rightarrow \infty} f_n d\mu = \int f d\mu.$$

Example: Let $f_n(x) = \frac{x}{n}$ and $E = \mathbb{R}_{\geq 0}$. Then each $f_n(x)$ is mble ($f^{-1}[a, \infty] = [na, \infty] \in \mathcal{B}$),
 $f_n \rightarrow 0$ μ -a.e., but

$$\int f_n d\mu = \int \frac{x}{n} dx = \frac{1}{n} \int_0^{\infty} x dx = \infty \quad \forall n$$

$$\Rightarrow \lim_{n \rightarrow \infty} \int f_n d\mu = \infty$$

$$\text{and } \int \lim_{n \rightarrow \infty} f_n d\mu = \int 0 d\mu = 0.$$