

5.3 Suppose $\{f_n\}$ are mble and suppose $\sum_{n=1}^{\infty} |f_n| \in L^1(\mu)$.
20/20 Notice that

$$\int \sum_{n=1}^{\infty} |f_n| d\mu \leq \int \left| \sum_{n=1}^{\infty} |f_n| \right| d\mu < \infty$$

and thus $\sum_{n=1}^{\infty} |f_n|$ converges μ -a.e. $\Rightarrow \sum_{n=1}^{\infty} f_n$ converges μ -a.e. Then $S_N = \sum_{n=1}^N f_n \rightarrow \sum_{n=1}^{\infty} f_n$ μ -a.e., $\{S_N\}$ are all mble (as sums of mble functions),
 $|S_N| = \left| \sum_{n=1}^N f_n \right| \leq \sum_{n=1}^N |f_n| \leq \sum_{n=1}^{\infty} |f_n|$ where $\sum_{n=1}^{\infty} |f_n|$ is integrable by hypothesis. Thus by the DCT,

$$\sum_{n=1}^{\infty} \int f_n d\mu = \lim_{N \rightarrow \infty} \sum_{n=1}^N \int f_n d\mu = \lim_{N \rightarrow \infty} \int \sum_{n=1}^N f_n d\mu$$

$$= \lim_{N \rightarrow \infty} \int S_N d\mu$$

DCT $\rightarrow$

$$= \int \lim_{N \rightarrow \infty} S_N d\mu$$

$$= \int \lim_{N \rightarrow \infty} \sum_{n=1}^N f_n d\mu = \int \sum_{n=1}^{\infty} f_n d\mu$$