

4.4 cont.

However, if $L(x) = U(x)$ then $L(x) = f(x) = U(x)$ m. a. e., which

implies that f is measurable, being a limit of measurable

functions (by 3.3.1 Lemma Strock p. 50). Hence

$$\int_{[0,1]} f dx = \int_{[0,1]} L(x) = \int_{[0,1]} f(x) = \int_{[0,1]} U(x) = \int_{[0,1]} f dx$$

Since f is integrable, $\int_{[0,1]} f dx = \int_{[0,1]} f dx = \int_{[0,1]} f dx = \int_{[0,1]} f d\mu$