

4.3 cont.

4.3 $\{\phi_m\}_{m=1}^{\infty}$ converges monotonically to f ($\phi_m \uparrow f$)

and by induction on the linearity property for measurable fns.

we have $\int \phi_m dm = \sum_{n=1}^m \int f_n dm$. The natural

application of the monotone convergence theorem shows:

$$\int f dm = \lim_{M \rightarrow \infty} \int \phi_M = \lim_{M \rightarrow \infty} \sum_{n=1}^M \int f_n dm = \sum_{n=1}^{\infty} \int f_n dm$$

(2) clearly follows from the lemma, because

$$\int_E f \cdot \mathbb{1}_{\{\cup_{n=1}^{\infty} \Gamma_n\}} dm = \sum_{n=1}^{\infty} \int_E f \cdot \mathbb{1}_{\Gamma_n} dm = \sum_{n=1}^{\infty} \int \mathbb{1}_{\Gamma_n} dm = \sum_{n=1}^{\infty} \nu(\Gamma_n)$$

The result holds if $\int_E f dm = \infty$ because no

assumptions were made about the finiteness of

$\int_E f dm$. f was assumed measurable, and non-negative, not integrable.