

By monotonicity of the integral for nondecreasing ψ , we then have:

$$\begin{aligned}\int_J fg \, d\psi &\leq \frac{1}{p} \int_J f^p \, d\psi + \frac{1}{q} \int_J g^q \, d\psi \\ &= \frac{1}{p} + \frac{1}{q} \\ &= 1 \\ &= \left(\int_J f^p \, d\psi \right)^{\frac{1}{p}} \left(\int_J g^q \, d\psi \right)^{\frac{1}{q}}\end{aligned}$$