

By using the norm from $X \oplus Y$, we can consider $\Gamma(\Lambda)$ as a closed subspace of $X \oplus Y$. If (x_n, y_n) is Cauchy in $\Gamma(\Lambda)$, it converges to some $(x, y) \in X \oplus Y$, but because $\Gamma(\Lambda)$ is closed, it must be that $(x, y) \in \Gamma(\Lambda)$. So $\Gamma(\Lambda)$ is complete and hence a Banach space.

Let $L : \Gamma(\Lambda) \rightarrow X$ be the bijective linear map $(x, \Lambda x) \mapsto x$; this map is continuous since it is a projection. Hence L is bounded, and therefore $L^{-1} : x \mapsto (x, \Lambda x)$ is also bounded, and hence continuous. Consequently, $x \mapsto \Lambda x$ is continuous, as desired.