

HOMEWORK 5 FOR 18.125, SPRING 2010
DUE THURSDAY, MARCH 11 AT THE BEGINNING OF
LECTURE.

HW5.1 Stroock 3.3.17.

HW5.2 Stroock 3.3.20.

HW5.3 Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions on a measure space $(E, \mathcal{B}, \mu)$ and assume

$$\sum_{n=1}^{\infty} |f_n|$$

is integrable. Prove that $\sum_{n=1}^{\infty} f_n$ is integrable and

$$\sum_{n=1}^{\infty} \int f_n d\mu = \int \sum_{n=1}^{\infty} f_n d\mu.$$

HW5.4 Suppose $\{f_n\}_{n=1}^{\infty}$ is a sequence of non-negative measurable functions on a measure space $(E, \mathcal{B}, \mu)$, and assume $f_n \searrow f$ for each $x \in E$. Assume further that $f_1 \in L^1(\mu)$. Prove

$$\lim_{n \rightarrow \infty} \int f_n d\mu = \int f d\mu.$$

Give an example to show this does not hold if $f \in L^1(\mu)$ is omitted from the assumptions.

HW5.5 Last week you showed if f is a non-negative measurable function on a measure space $(E, \mathcal{B}, \mu)$ then

$$\nu(\Gamma) := \int_{\Gamma} f d\mu, \quad \Gamma \in \mathcal{B}$$

is a measure on $(E, \mathcal{B})$. Let g be a non-negative measurable function on E and show that

$$\int g d\nu = \int g f d\mu.$$

(*Hint:* First show for g a simple function and use Monotone Convergence Theorem.)